System Identification of A Drone

Name: David (王琮文)
Student ID: R09522848
National Taiwan University
June 02, 2022

Contents

1 Introduction

Inner-loop control

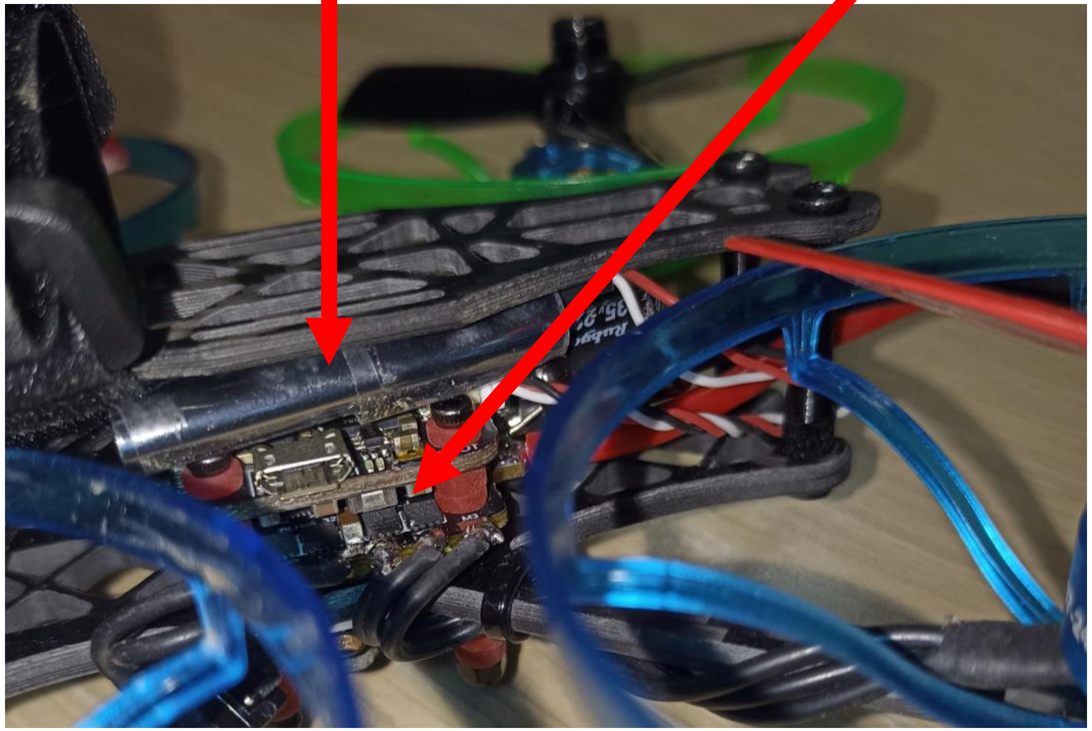
Drone and remote control

Data Acquisition

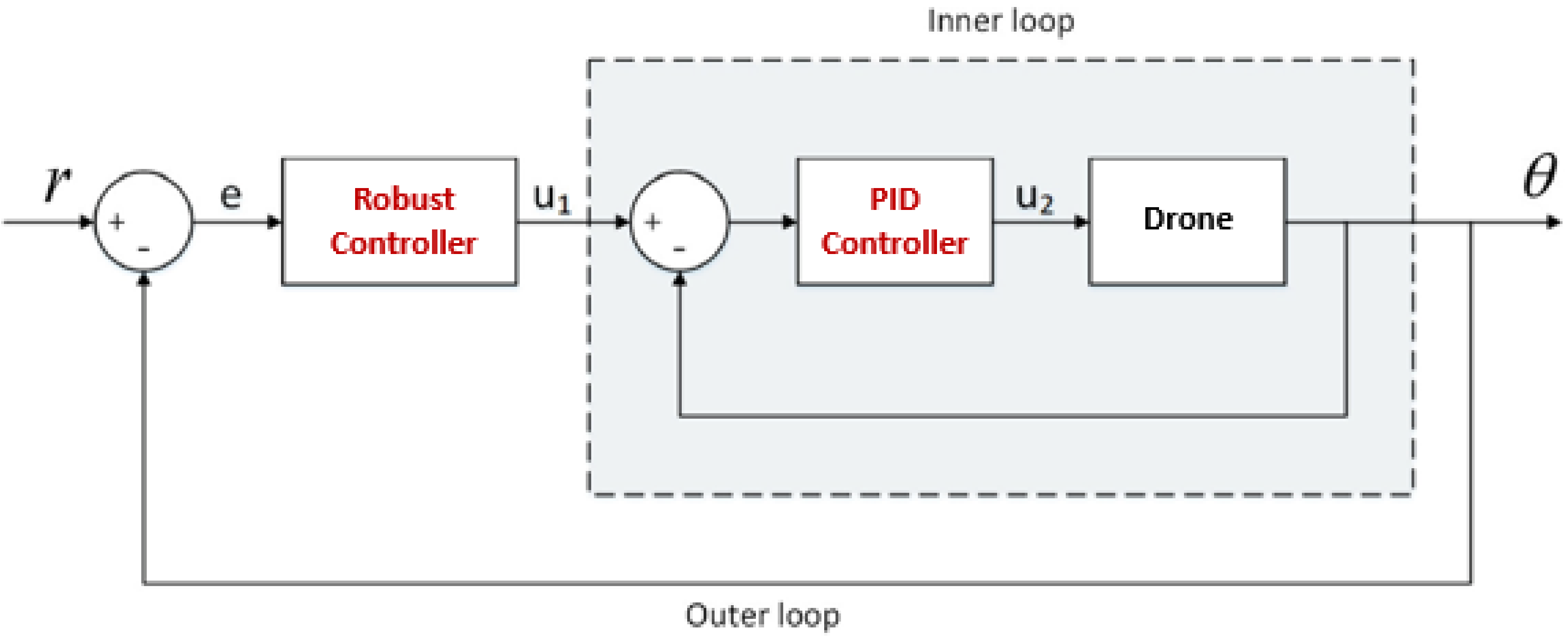
ToF Range Finder

IMU Sensor x2

SBUS (Receiver)



Controls



Inner loop
r
+
-
e
Robust Controller
u_1
+
-
PID Controller
u_2
Drone
θ
Outer loop

PID Control

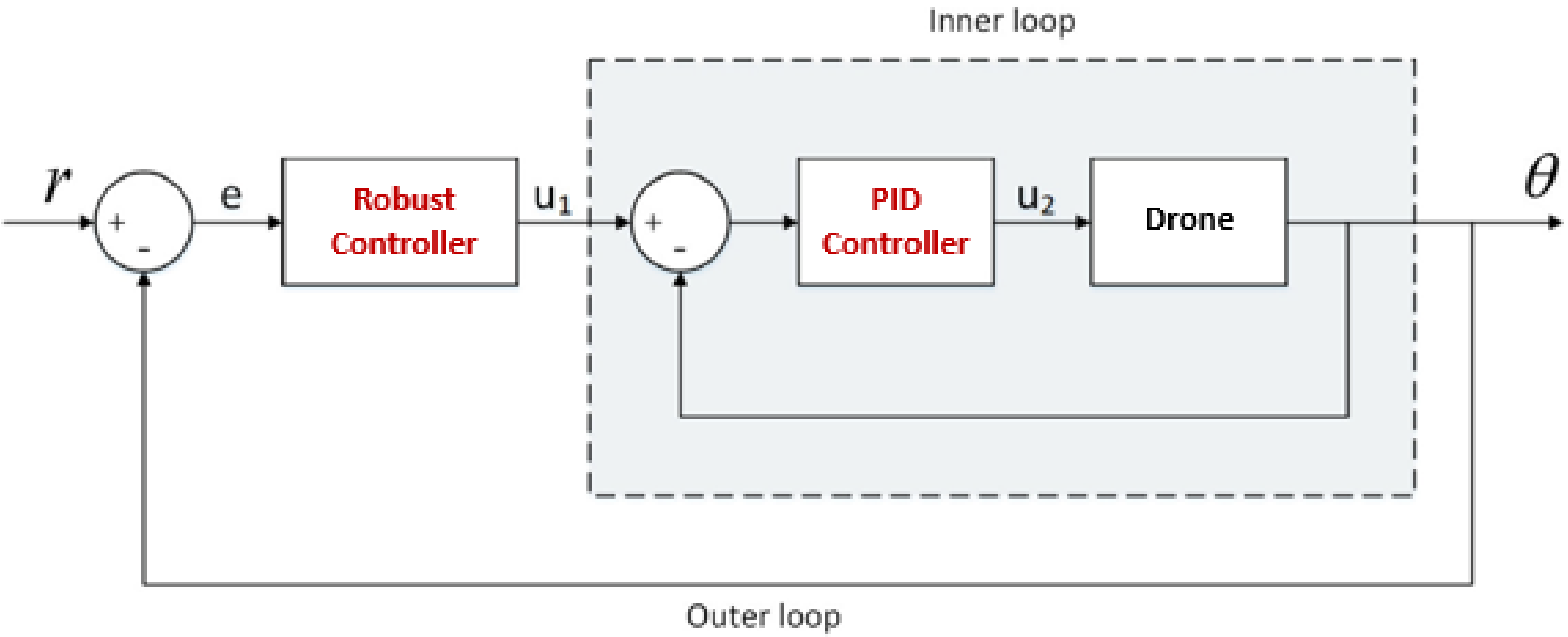
- 4 sets of PID Control:
  1. Roll
  2. Pitch
  3. Yaw
  4. Height
- Each set has two cascaded PID controllers:
  for example: PID control for roll ϕ and roll rate $\dot{\phi}$

PID Control

Speed 1x

2 Closed-loop System Identification

Problem Statement



System Identification

- However, yaw is estimated simply by two IMU sensors, so only a **3-input 3-output** system is considered.

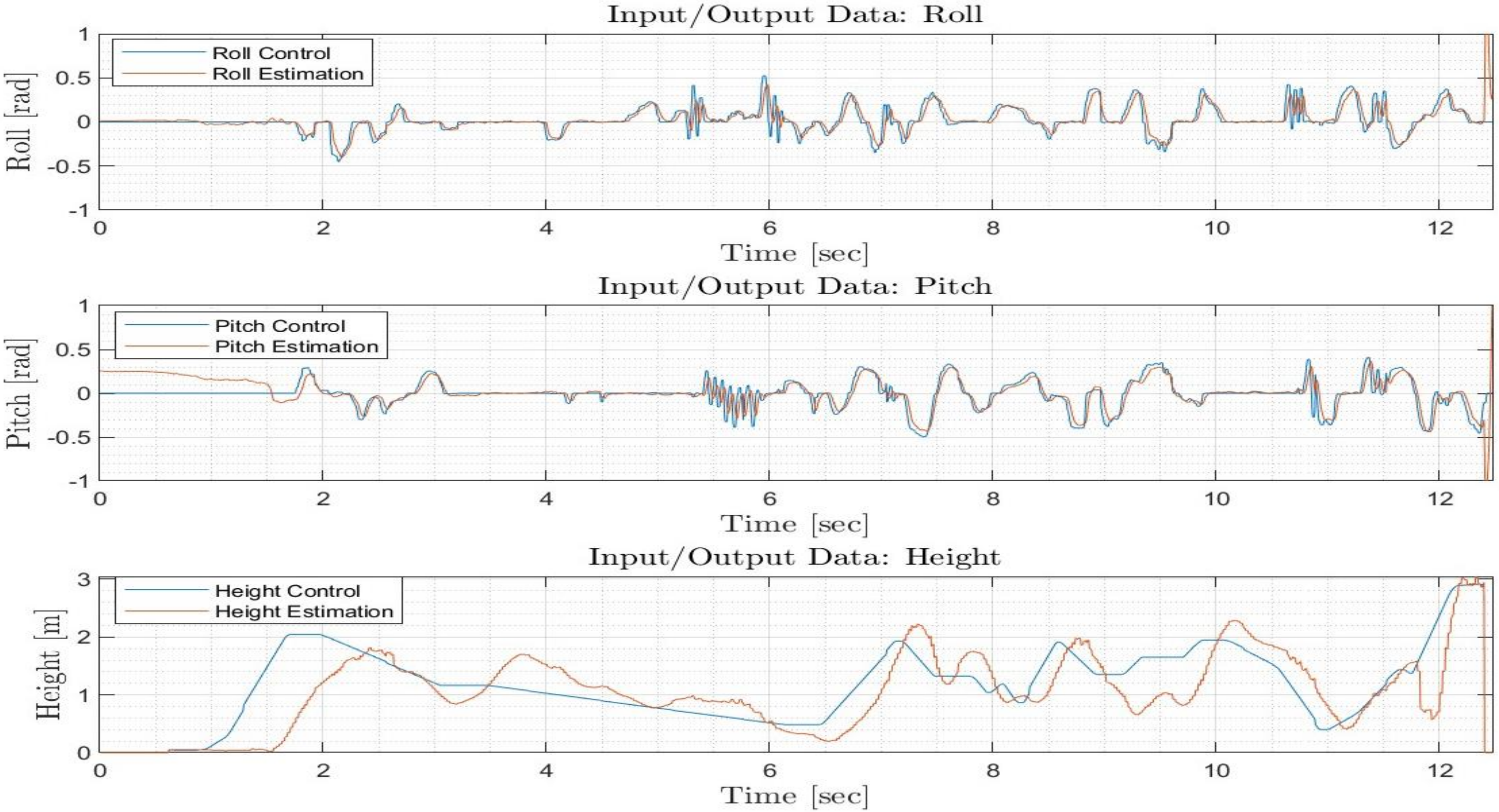
- Control Input (from a remote control): Roll, Pitch, Height
- System Output: Roll, Pitch, Height
- Sample time: 0.005 seconds

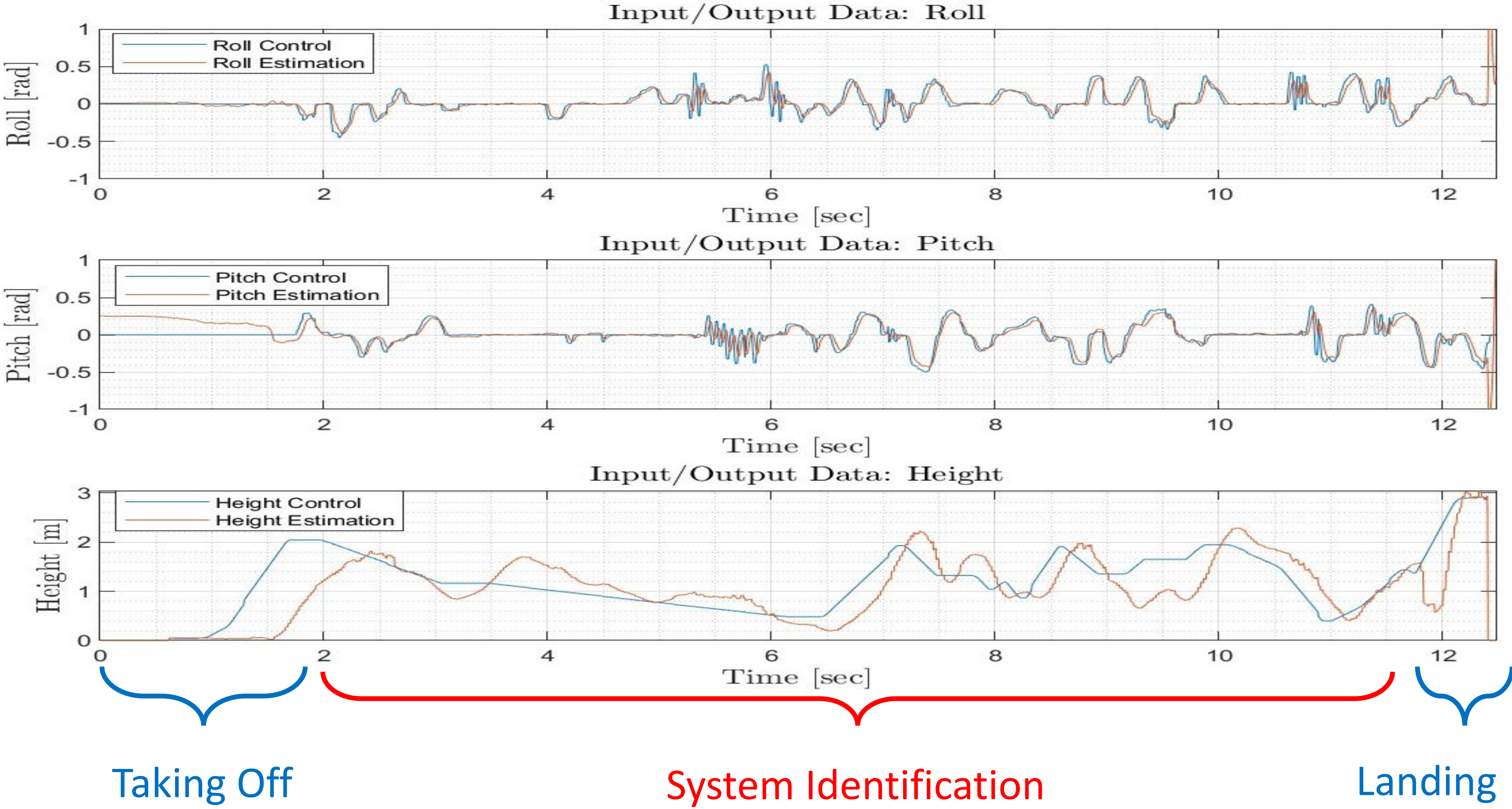
System Identification

- Input-Output Data
- MISO System Identification
- MIMO System Identification

Input-Output Data

Speed 2x


Input/Output Data: Roll
Roll Control
Roll Estimation
Roll [rad]
Time [sec]
Input/Output Data: Pitch
Pitch Control
Pitch Estimation
Pitch [rad]
Time [sec]
Input/Output Data: Height
Height Control
Height Estimation
Height [m]
Time [sec]


Input/Output Data: Roll
Roll Control
Roll Estimation
Roll [rad]
Time [sec]
Input/Output Data: Pitch
Pitch Control
Pitch Estimation
Pitch [rad]
Time [sec]
Input/Output Data: Height
Height Control
Height Estimation
Height [m]
Time [sec]
Taking Off
System Identification
Landing

System Identification

- Input-Output Data
- MISO System Identification
- MIMO System Identification

MISO System Identification

Case 1

- Control Input (from a remote control): Roll, Pitch, Height
- System Output: Roll

Case 2

- Control Input (from a remote control): Roll, Pitch, Height
- System Output: Pitch

Case 3

- Control Input (from a remote control): Roll, Pitch, Height
- System Output: Height

MISO System Identification

Case 1

- Control Input: Roll, Pitch, Height
- System Output: Roll

Note:

AIC: To minimize $\left\{ \log(V) + \frac{2d}{N} \right\}$

where V is the loss function, d is the total number of parameters in the structure in question, and N is the number of data points used for the estimation

- Procedure

1) Compute **loss functions V** for single-output ARX models.
2) Compute the normalized **Akaike's Information Criterion (AIC)** value.
3) Select **model order** for single-output ARX models.
4) Estimate the **ARX Model** by using least squares.

MISO System Identification: Case 1

- Control Input: Roll, Pitch, Height
- System Output: Roll

- Discrete-time ARX model: $A(z)y(t) = B(z)u(t) + e(t)$

  $A(z) = 1 - 1.202\ z^{-1} + 0.3244\ z^{-2} + 0.1292\ z^{-3} - 0.1059\ z^{-4}$

  $B1(z) = 0.06479 - 0.07467\ z^{-1} + 0.1456\ z^{-2}$

  $B2(z) = 0.02157 - 0.01912\ z^{-1}$

  $B3(z) = -0.4842 + 0.8863\ z^{-1} - 0.4018\ z^{-2}$

- Fit to estimation data: 94.1%
- Akaike’s Final Prediction Error: 5.697e-05

MISO System Identification: Case 2

- Control Input: Roll, Pitch, Height
- System Output: Pitch

- Discrete-time ARX model: $A(z)y(t) = B(z)u(t) + e(t)$

  $A(z) = 1 - 1.106 z^{-1} + 0.1053 z^{-2} + 0.1307 z^{-3}$

  $B1(z) = -0.001201 + 0.005909 z^{-1}$

  $B2(z) = 0.07366 z^{-1} + 0.04504 z^{-2}$

  $B3(z) = -0.1496 z^{-1} + 0.149 z^{-2}$

- Fit to estimation data: 94.4%
- Akaike's Final Prediction Error: 6.356e-05

MISO System Identification: Case 3

- Control Input: Roll, Pitch, Height
- System Output: Height

- Discrete-time ARX model: $A(z)y(t) = B(z)u(t) + e(t)$

  $A(z) = 1 - 0.9646 z^{-1} + 0.00499 z^{-2} - 0.02892 z^{-3}$

  $B1(z) = -0.0008751 z^{-2} - 0.09193 z^{-3} + 0.07976 z^{-4}$

  $B2(z) = -0.0563 + 0.06576 z^{-1}$

  $B3(z) = -0.162 z^{-2} + 0.1737 z^{-3}$

- Fit to estimation data: 94.0%
- Akaike's Final Prediction Error: 0.0008837

System Identification

- Input-Output Data
- MISO System Identification
- MIMO System Identification

Estimation Model

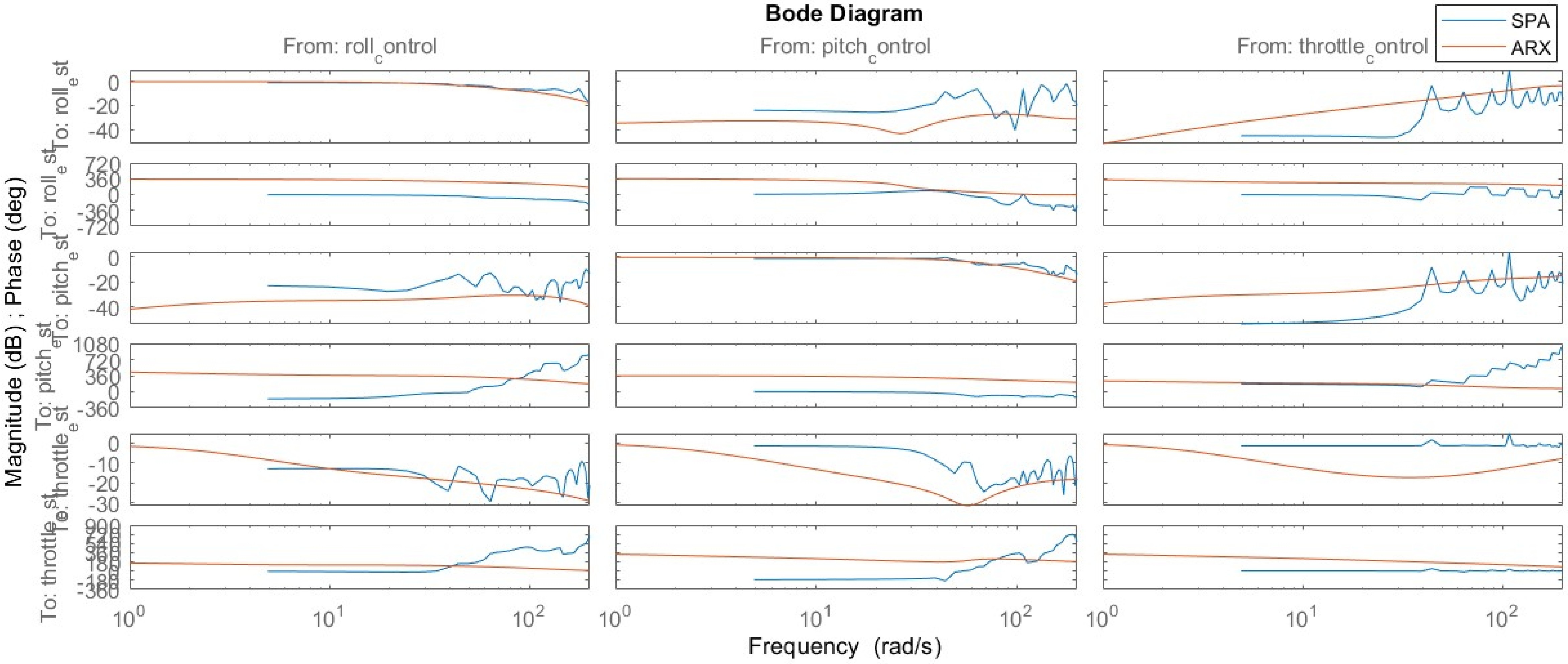
1) Estimate parameters of **ARX** model.
2) Estimate parameters of **ARMAX** model using time-domain data.
3) Estimate **Box-Jenkins (BJ)** polynomial model using time domain data.
4) **ARX** model estimation using four-stage instrumental variable method (**iv4**).
5) Estimate **output-error (OE)** polynomial model using time-domain data.
6) Estimate **state-space** model using **subspace** method with time-domain data.

Note: Assume the same order of the above model
order of A = 4, order of B = 3.

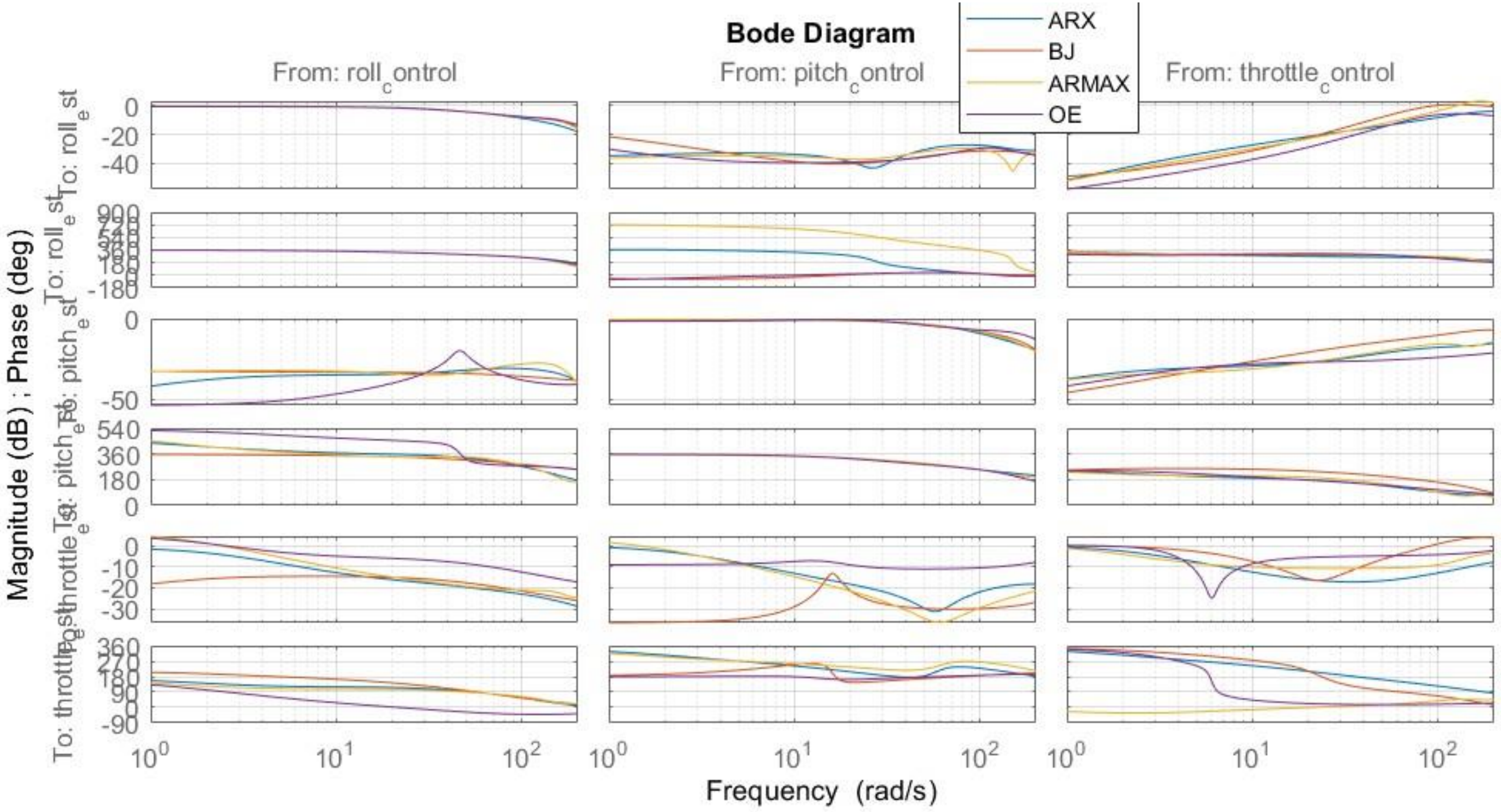
four-stage instrumental variable method

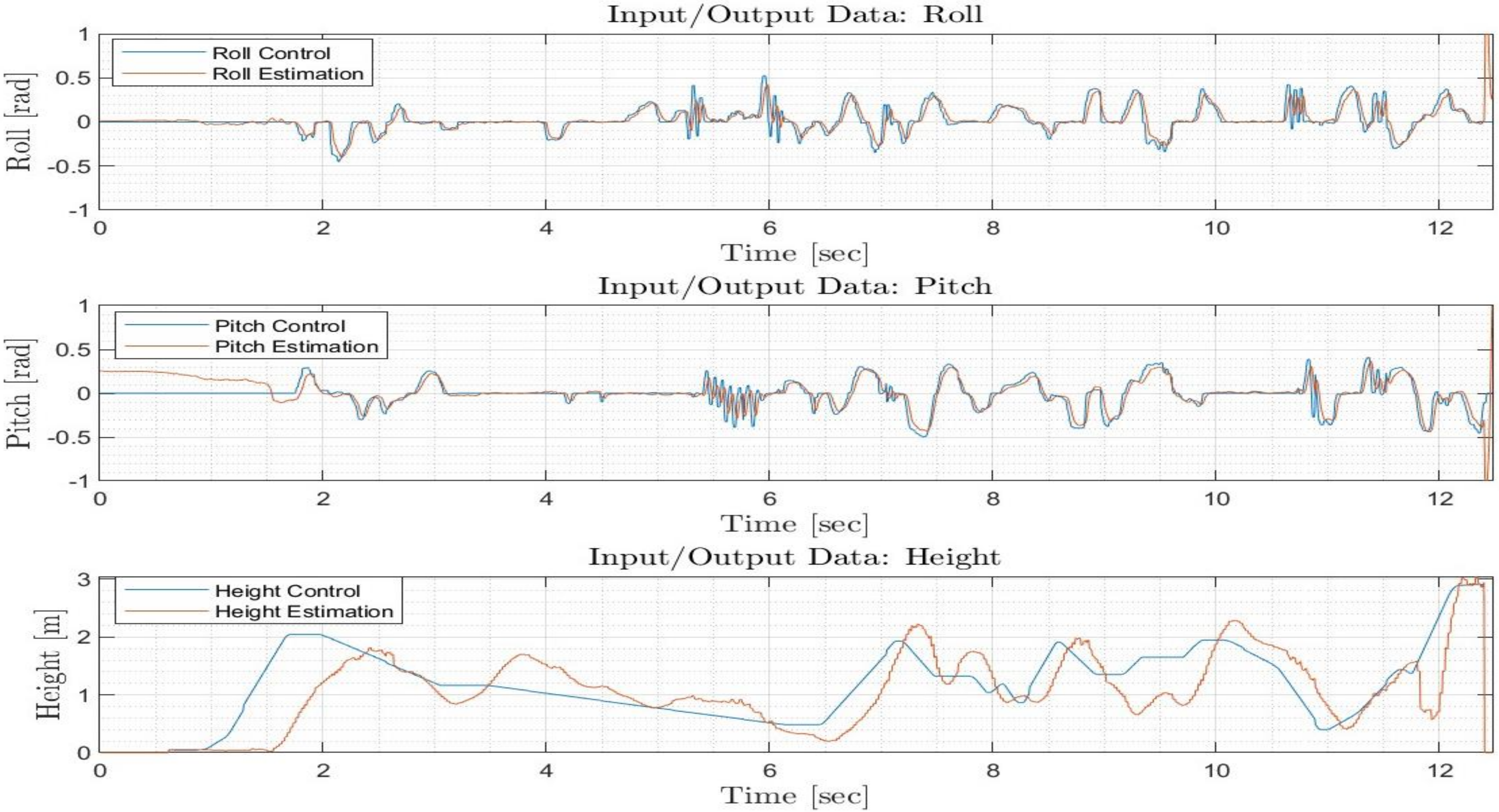
- The first stage uses the arx function.
- The resulting model generates the instruments for a second-stage IV estimate.
- The residuals obtained from this model are modeled as a high-order AR model.
- At the fourth stage, the input-output data is filtered through this AR model and then subjected to the IV function with the same instrument filters as in the second stage.

Frequency Response



Frequency Response




Input/Output Data: Roll
Roll Control
Roll Estimation
Roll [rad]
1
0.5
0
-0.5
-1
0
2
4
6
8
10
12
Time [sec]
Input/Output Data: Pitch
Pitch Control
Pitch Estimation
Pitch [rad]
1
0.5
0
-0.5
-1
0
2
4
6
8
10
12
Time [sec]
Input/Output Data: Height
Height Control
Height Estimation
Height [m]
3
2
1
0
0
2
4
6
8
10
12
Time [sec]

Transfer Function Coefficients

- Input = Roll
- Output = Roll
- Assume
  No Delay time

	b0	b1	b2	a0	a1	a2	a3	a4
ARX	0.06505	-0.07508	-0.145	1	- 1.211	0.3283	0.1454	-0.1193
BJ	0.06323	-0.09669	0.1436	1	-1.658	1.028	- 0.1213	- 0.1327
ARMAX	0.06759	- 0.1041	0.1448	1	-1.637	0.9694	-0.07756	-0.1405
OE	0.0486	-0.08014	0.1594	1	0.1594	0.7712	-0.0705	- 0.1056

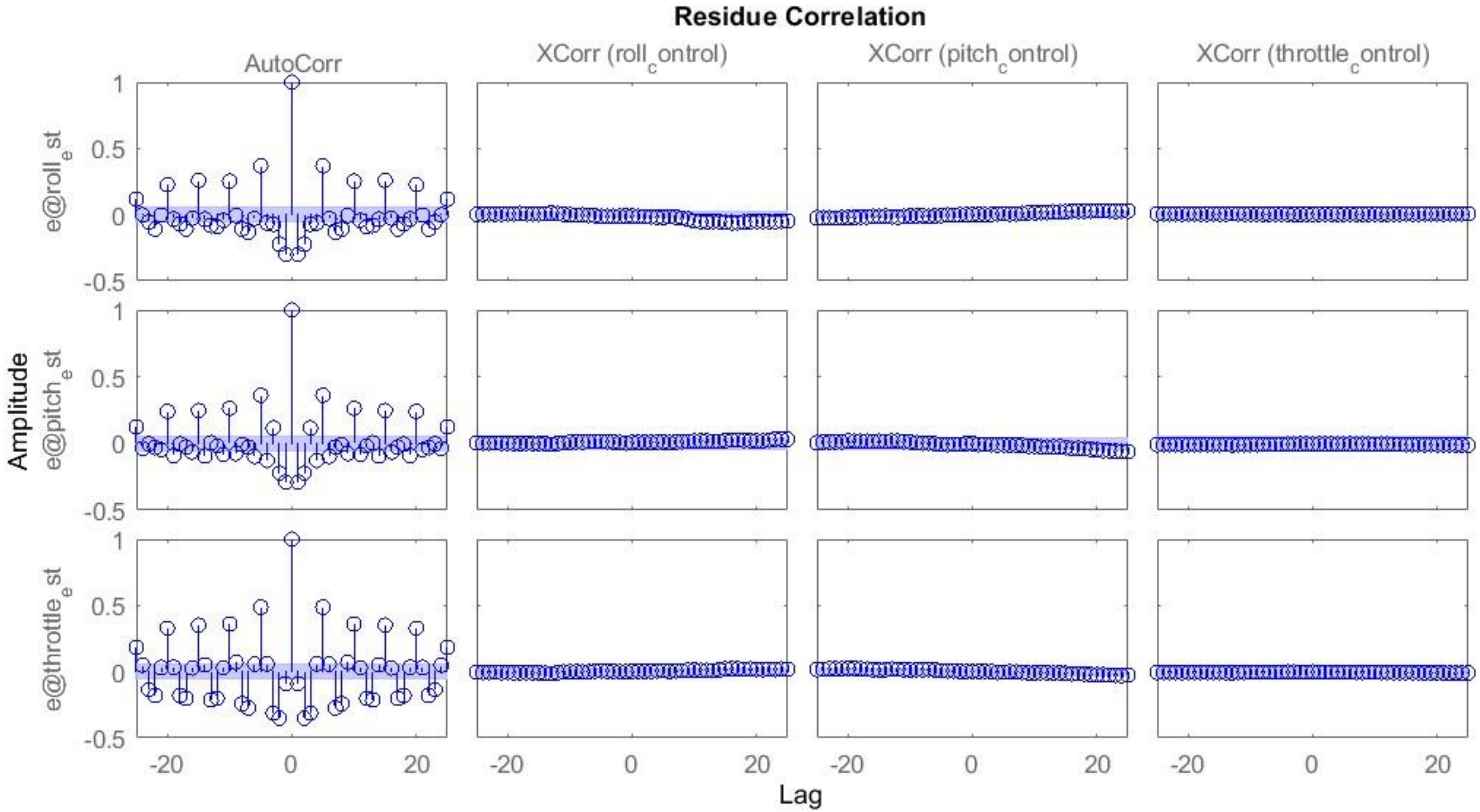
What Is Residual Analysis?

- **Residuals** are differences between the one-step-predicted output from the model and the measured output from the validation data set.
- According to the **whiteness test criteria**, a good model has the residual autocorrelation function inside the confidence interval of the corresponding estimates, indicating that the residuals are uncorrelated.
- According to the **independence test criteria**, a good model has residuals uncorrelated with past inputs. Evidence of correlation indicates that the model does not describe how part of the output relates to the corresponding input.

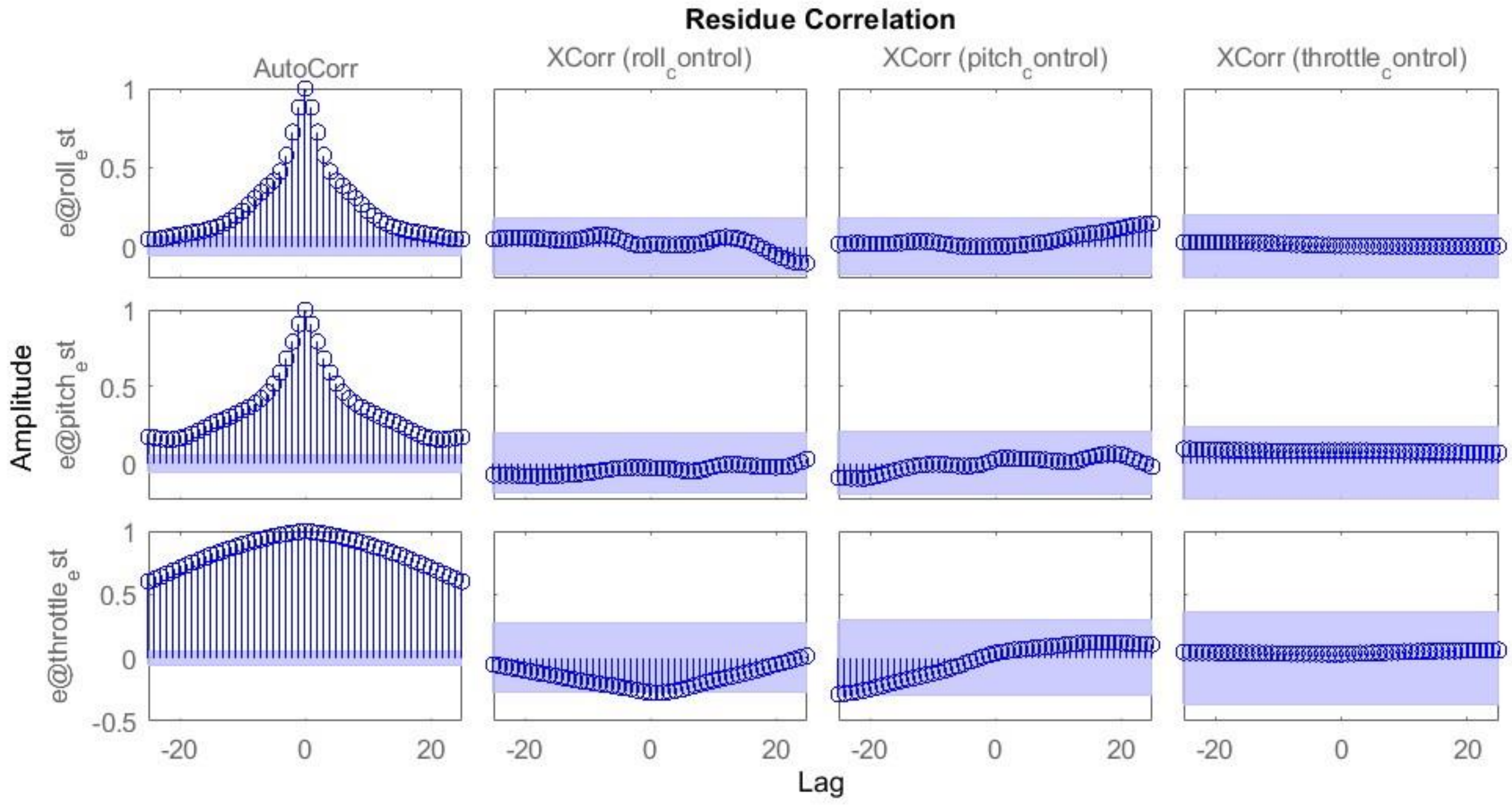
Confidence Interval

- The confidence interval corresponds to the range of residual values with a specific probability of being statistically insignificant for the system.
- For example, for a 99.7% confidence interval, the region around zero represents the range of residual values that have a 99.7% probability of being statistically insignificant.

Correlation Test for iv4



Correlation Test for OE

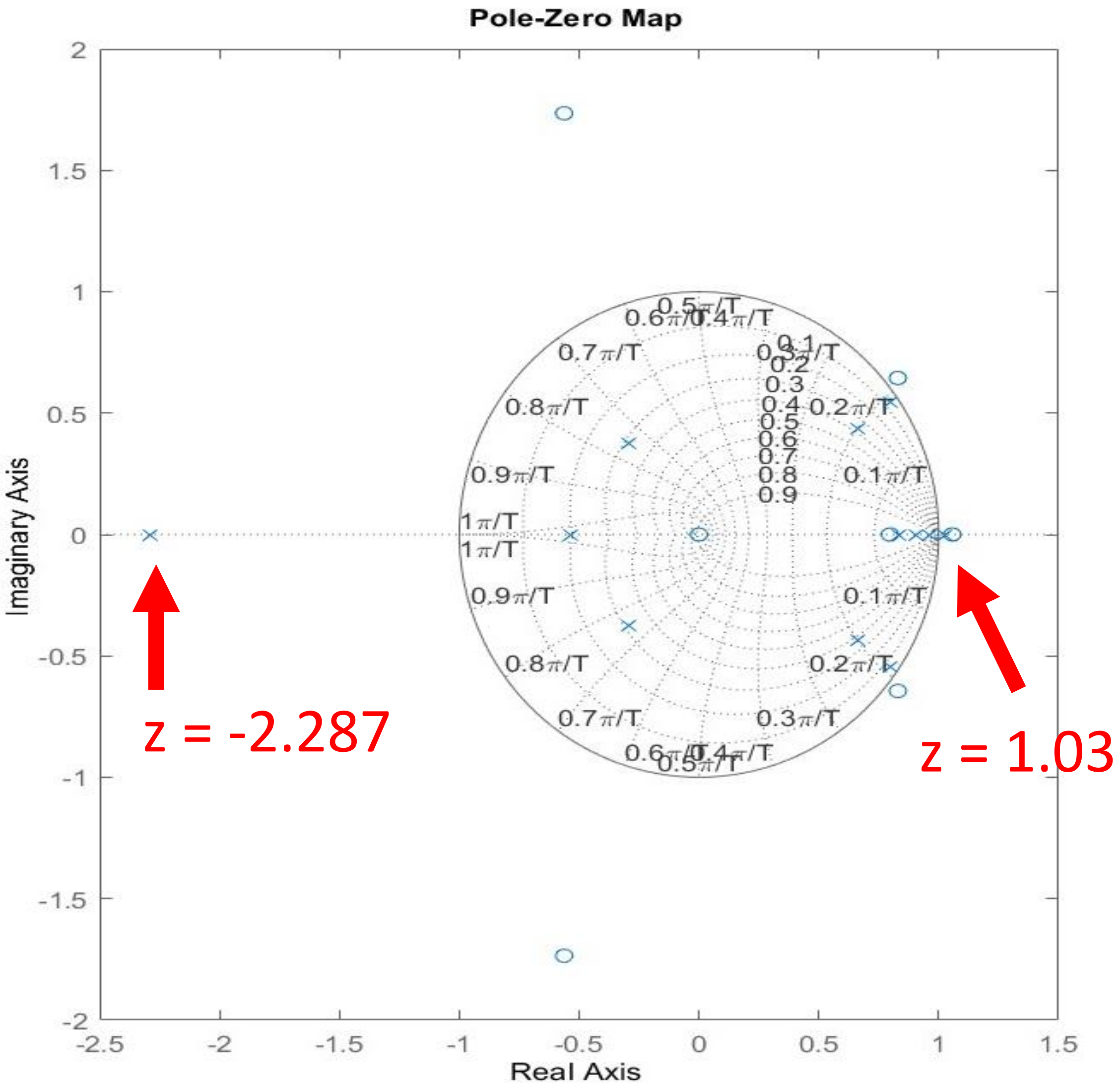


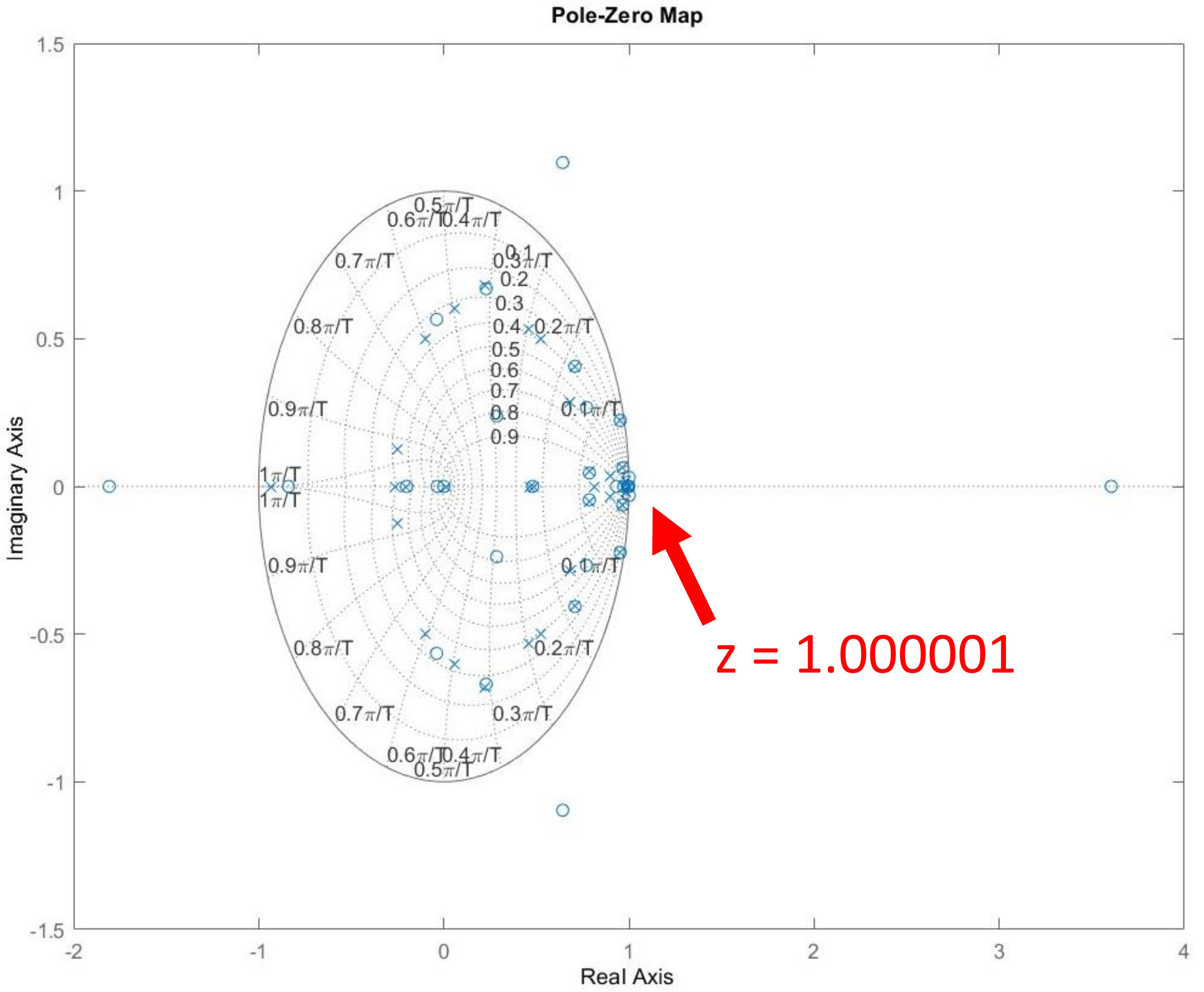
Unstable Models

- Unstable models: **iv4** and **OE** model
- The iv4 model has unstable poles at $z = -2.287, 1.03$
- The OE model has unstable poles at: $z = 1.000001$

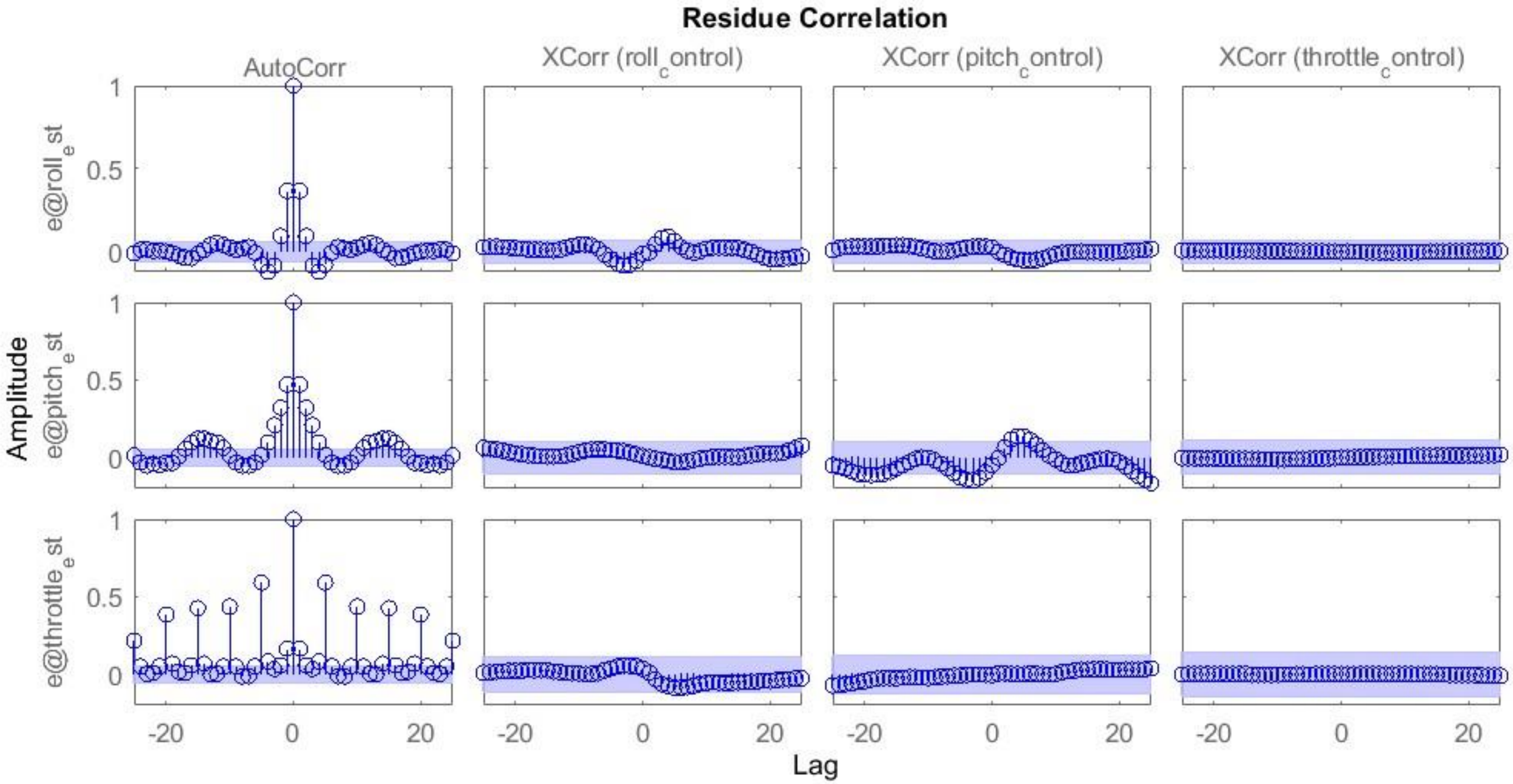
iv4 Model

OE Model

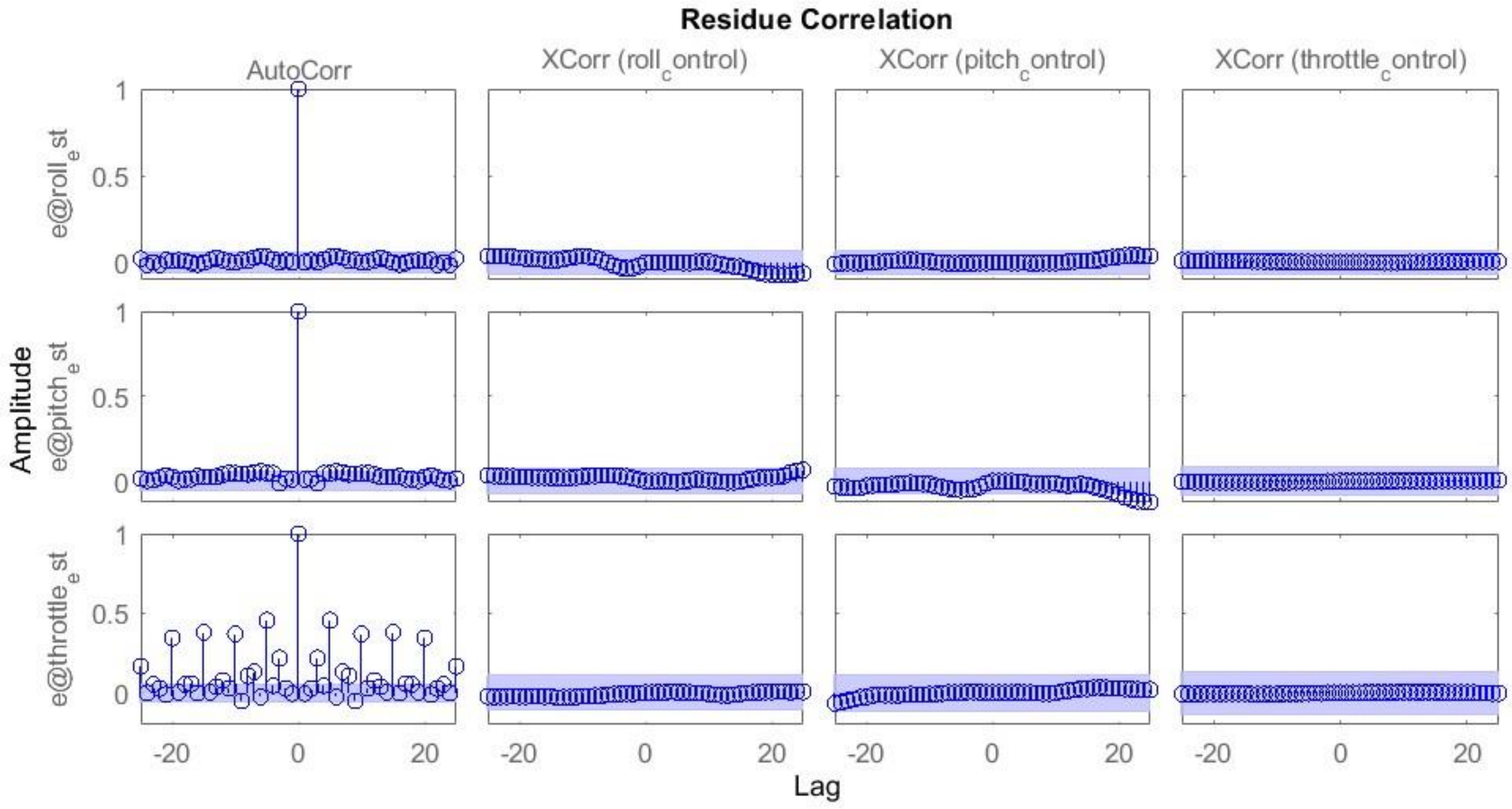




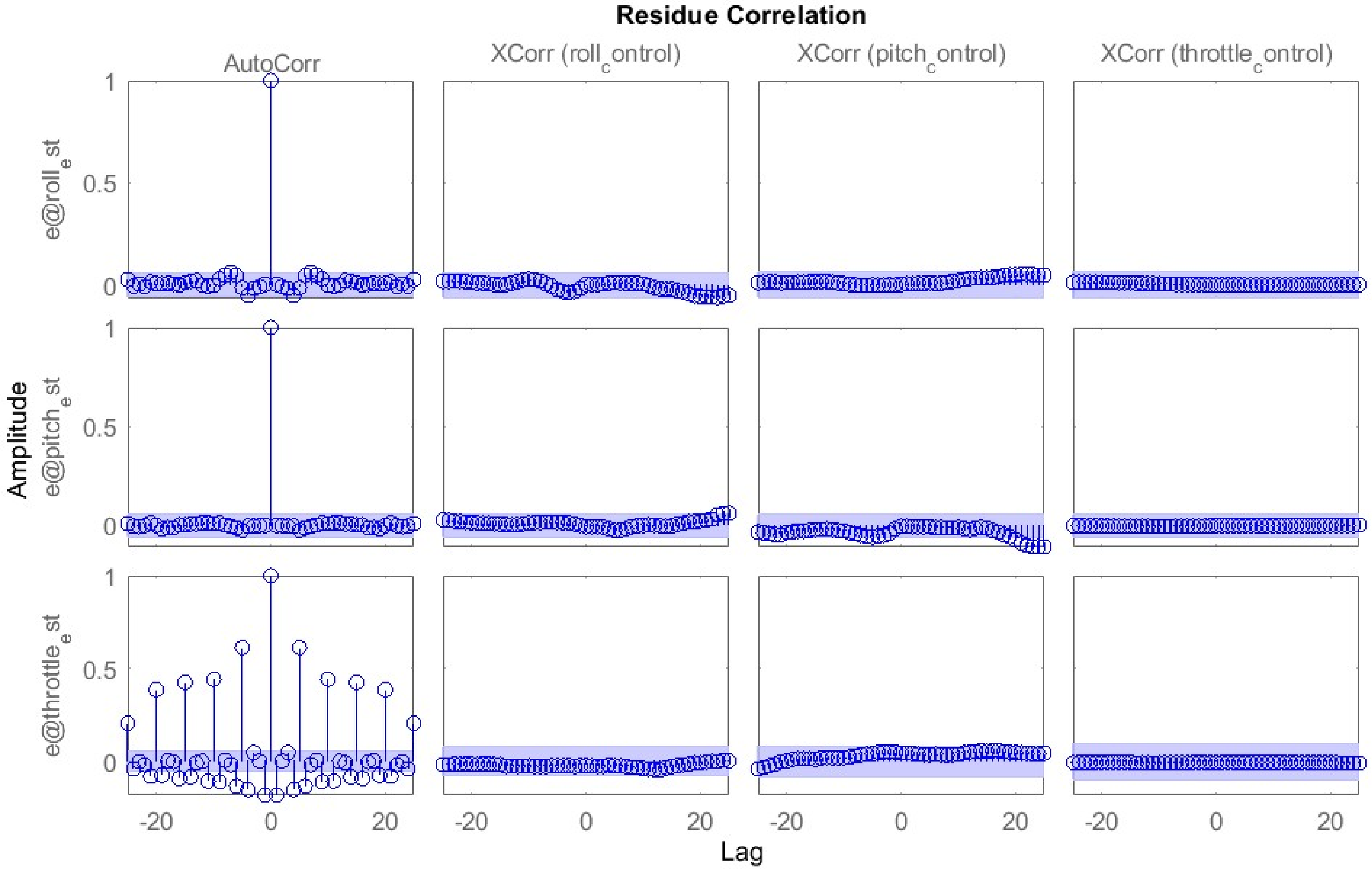
Correlation Test for n4sid



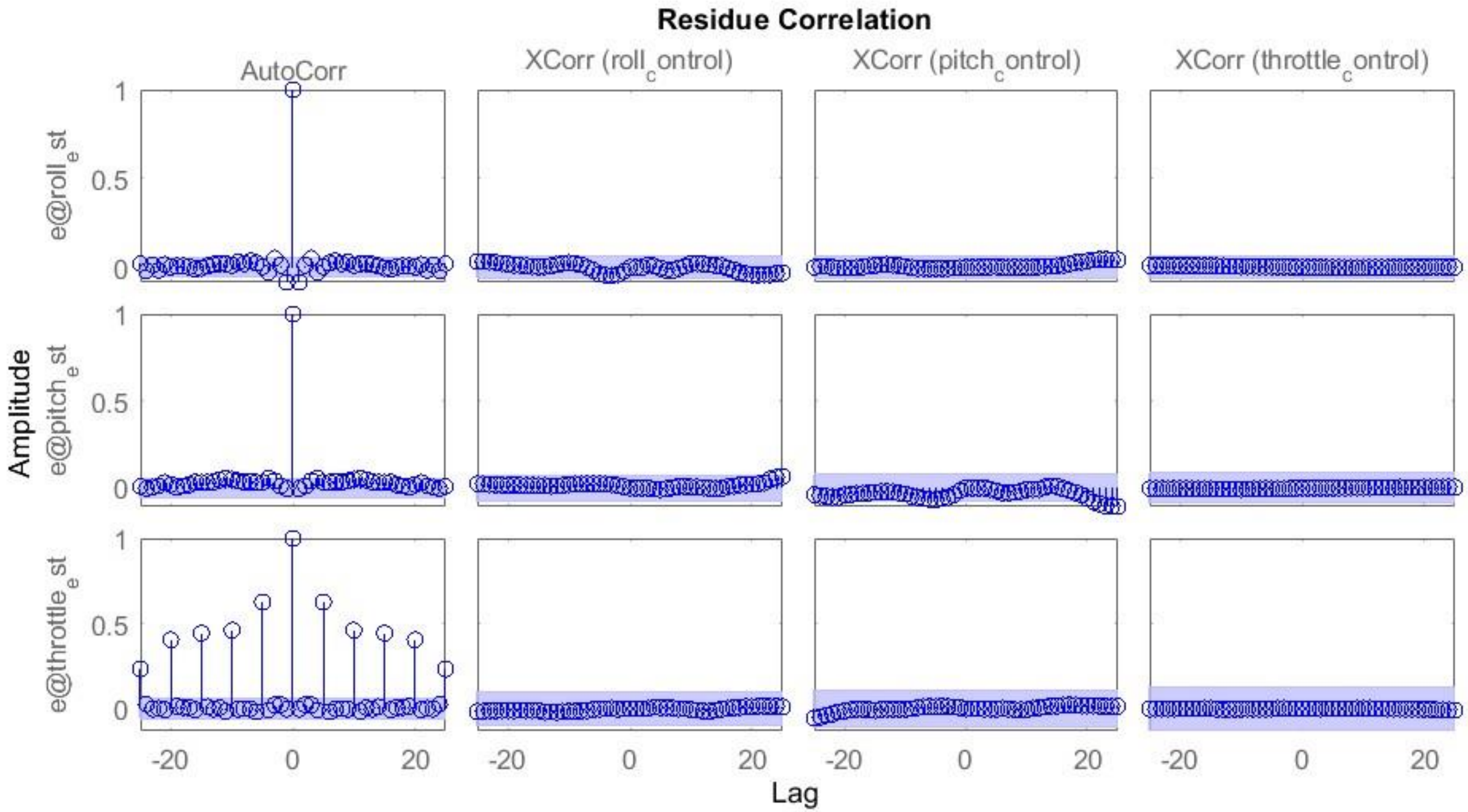
Correlation Test for ARMAX



Correlation Test for BJ



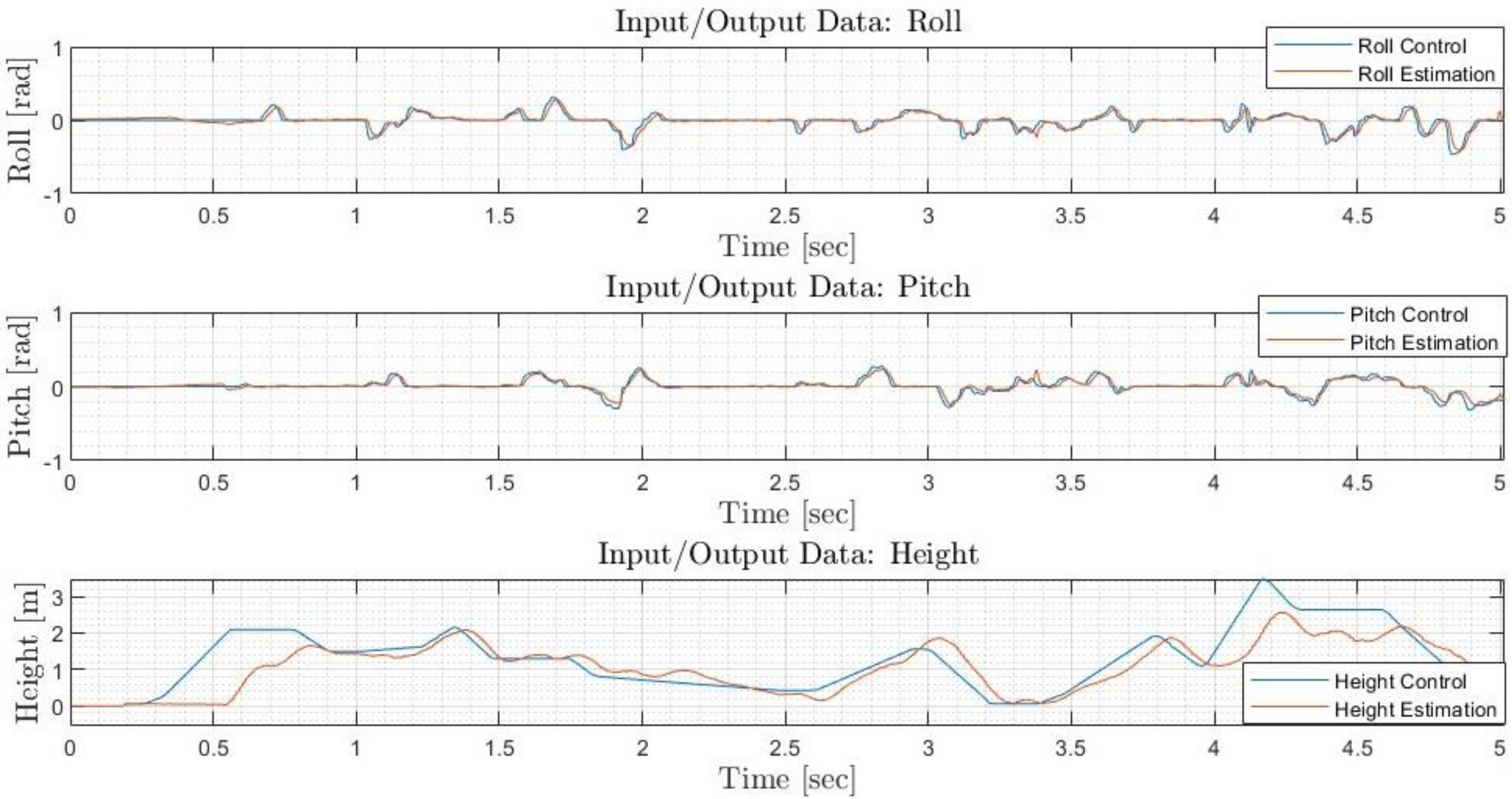
Correlation Test for ARX



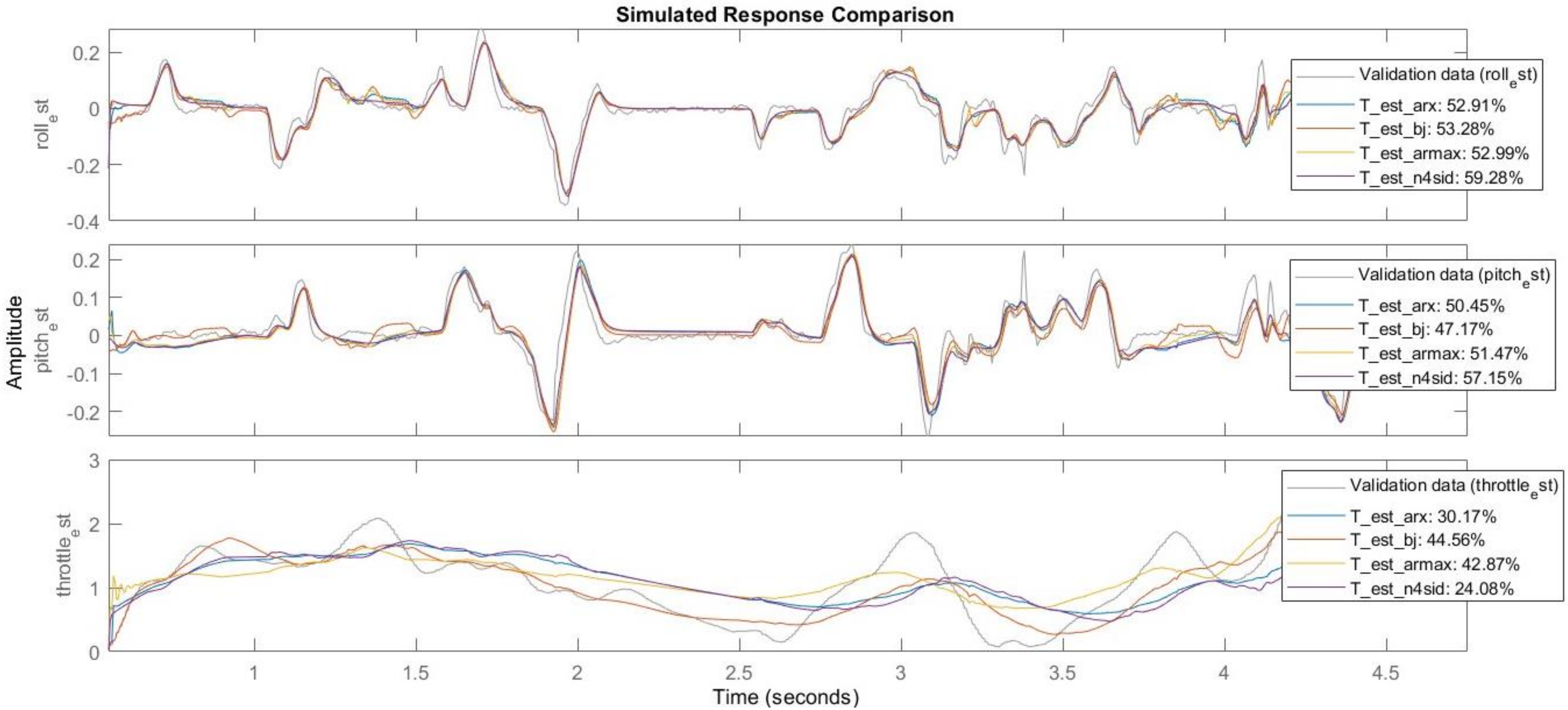
Discussion

- The ARX model is estimated using the **regularization** kernel.
- Estimate parameters of **ARX**, ARIX, AR, or ARI model.
- Estimate parameters of **ARMAX**, ARIMAX, ARMA, or ARIMA model using time-domain data.

Model Validation



Model Validation



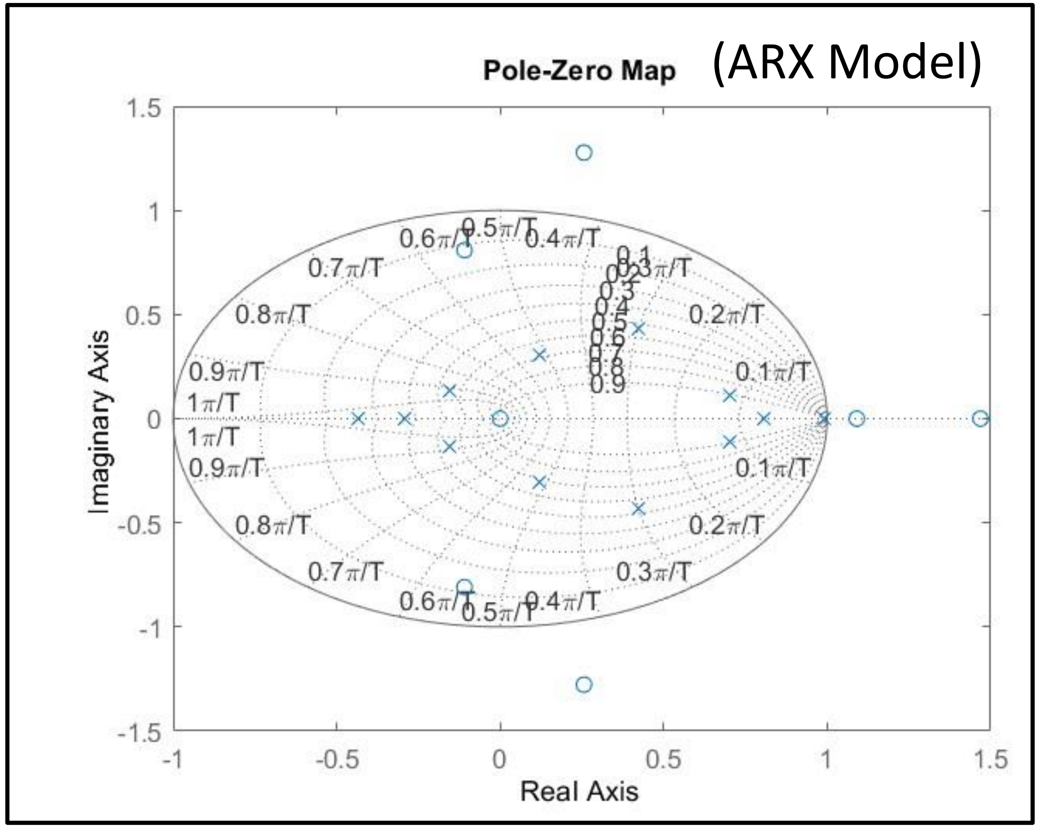
Conclusion

- Stable models (better results):

ARX, ARMAX, and BJ model,

and the state-space model (n4sid) is good, too.

- Unstable models: iv4 and OE model

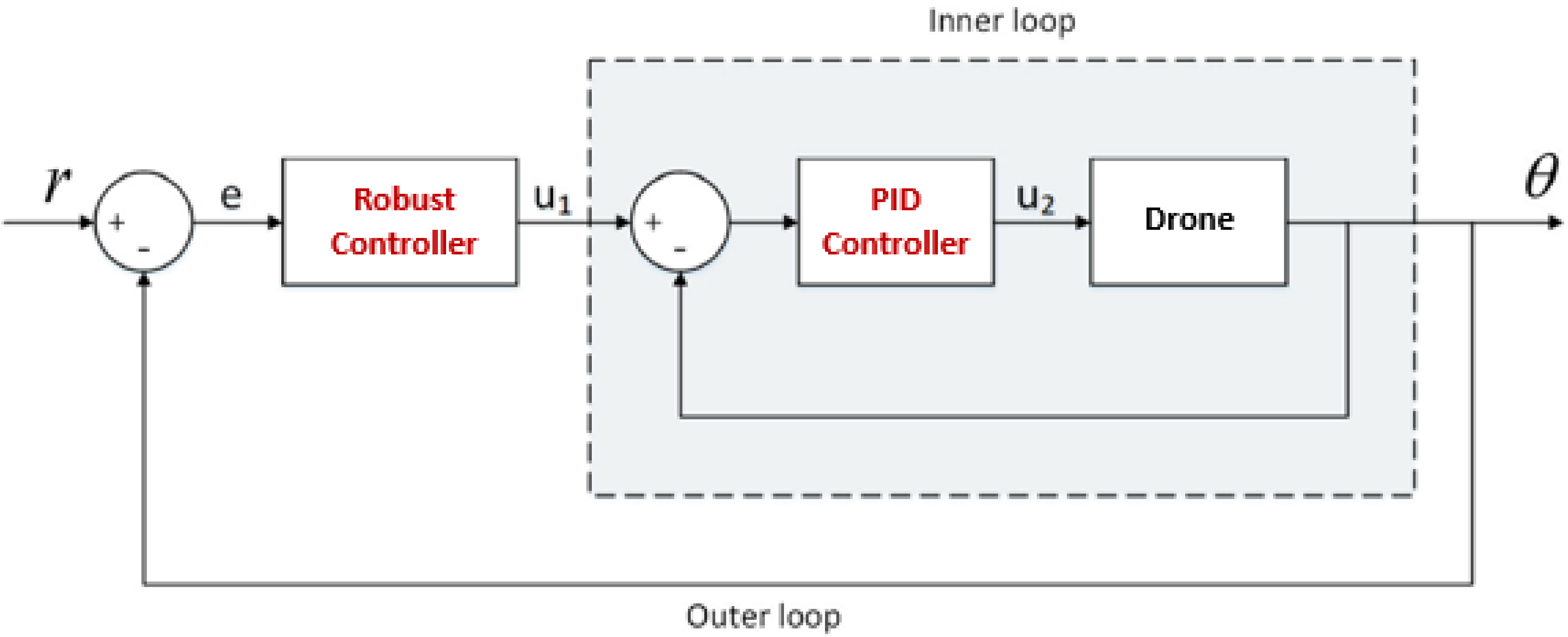


3

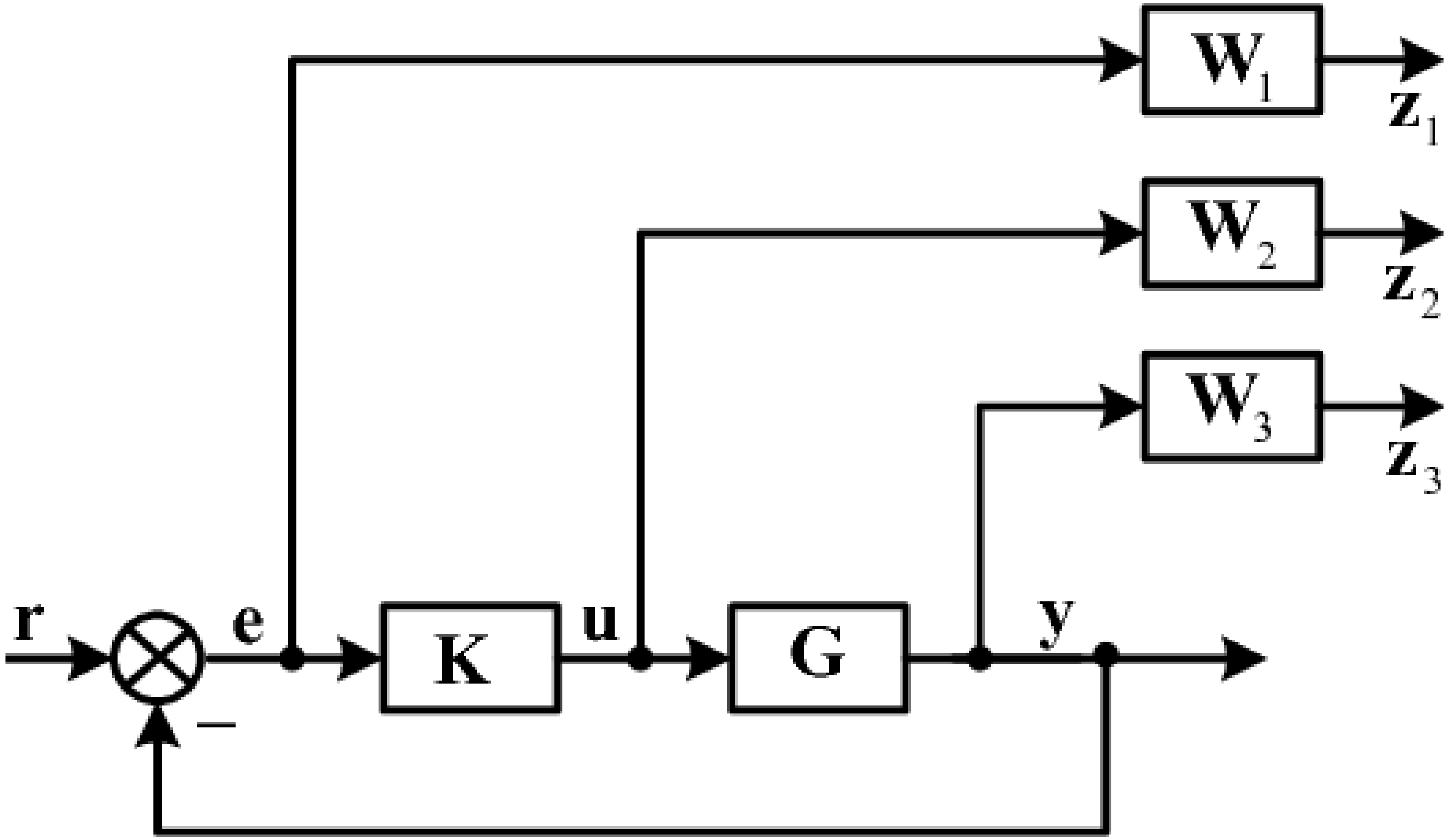
Design of Robust Controllers

Outer-loop control

Outer-loop Control



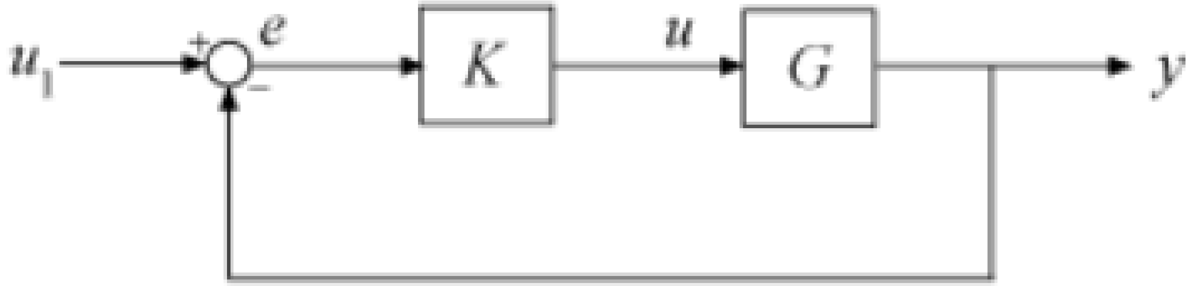
Mixed-sensitivity H∞ synthesis



Mixed-sensitivity H∞ synthesis

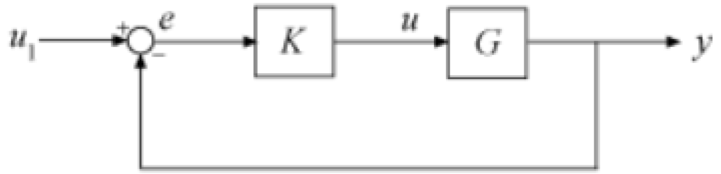
- Design a controller that minimizes the H_∞ norm of the weighted closed-loop transfer function M(s).

$$M(s) = \begin{bmatrix} W_1 S \\ W_2 KS \\ W_3 T \end{bmatrix},$$



Mixed-sensitivity H∞ synthesis

- For the controller K

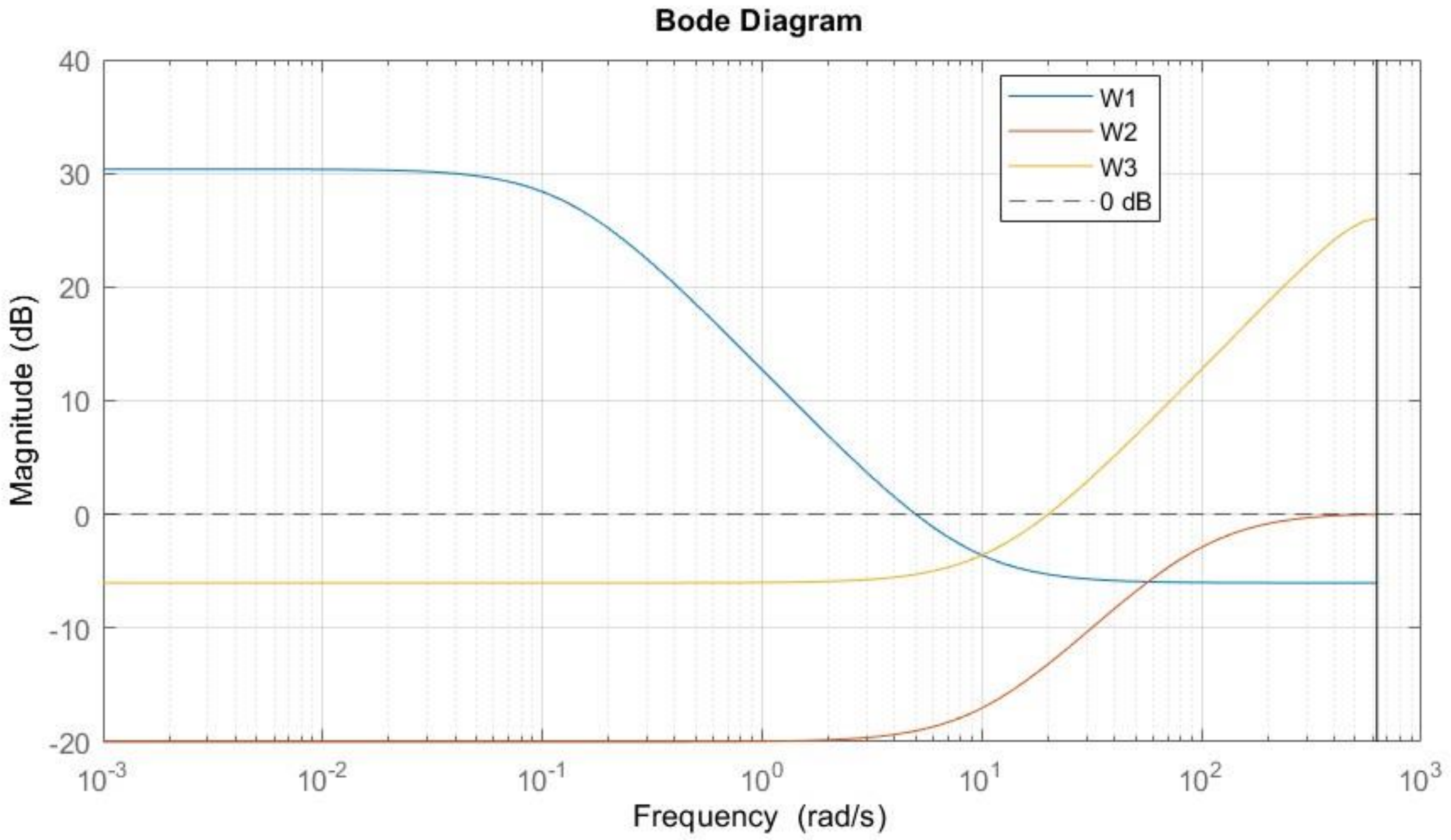


$$\|S\|_\infty \le \gamma |W_1^{-1}|$$

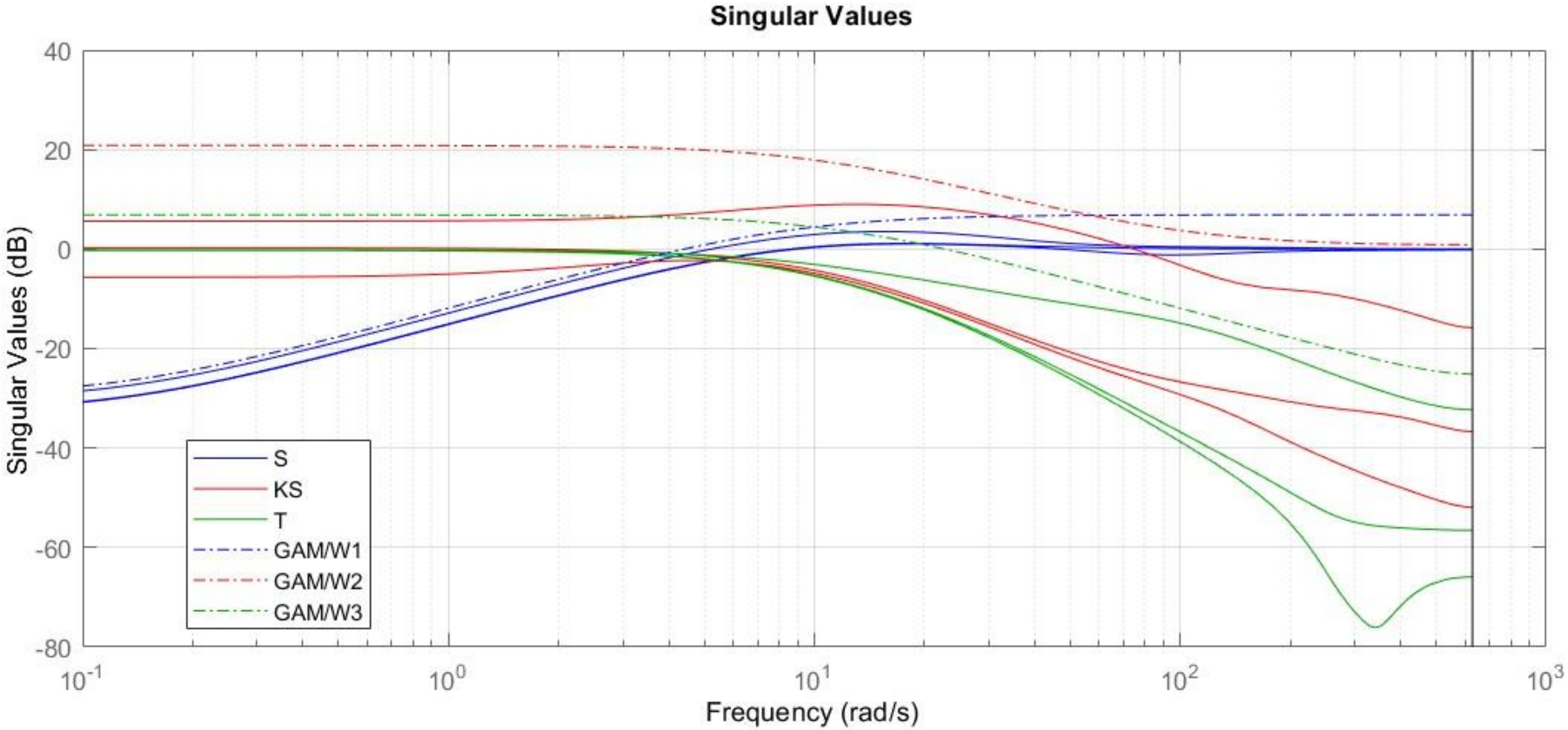
$$\|KS\|_\infty \le \gamma |W_2^{-1}|$$

$$\|T\|_\infty \le \gamma |W_3^{-1}|.$$

Mixed-sensitivity H∞ synthesis



Mixed-sensitivity H∞ synthesis




THANK YOU FOR LISTENING
TO MY PRESENTATION