Robust Control – Final Project

μ -synthesis Design of A Half-Car Active Suspension System

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Outlines

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μ -synthesis

02

Systems

Half-Car Model

Passenger Model

Hydraulic Actuator

03

Results

μ -synthesis

Structured Singular Value

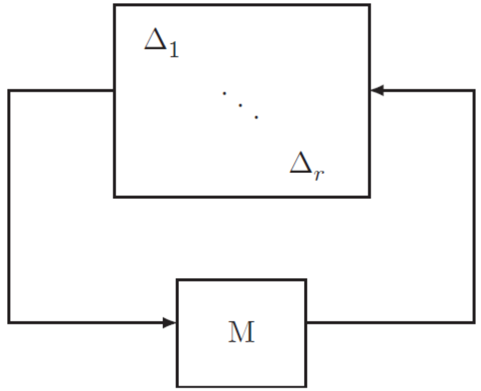


Figure 17.4 System with structured uncertainty

Definition 17.1 *For any given matrix* $M \in \mathbb{C}^{n\times n}$*, the structured singular value* $\mu_{\mathbf{\Delta}}(M)$ *is defined as*

$$\mu_{\mathbf{\Delta}}(M) = \frac{1}{\min\{\sigma_{\max}(\Delta) \mid \Delta \in \mathbf{\Delta},\ \det(I - M\Delta) = 0\}}. \tag{17.10}$$

$\mu_{\mathbf{\Delta}}(M) = 0$ *when there is no* $\Delta \in \mathbf{\Delta}$ *satisfying* $\det(I - M\Delta) = 0$.

Structured Singular Value

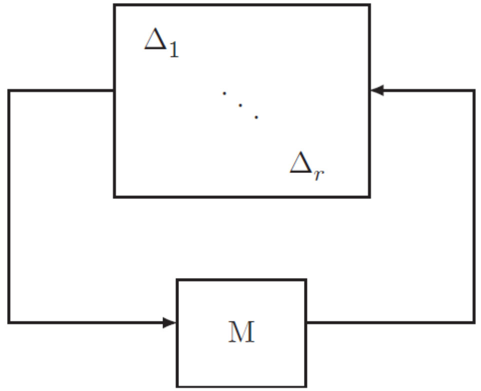


Figure 17.4 System with structured uncertainty

17.3.1.1 Single Scalar Block Uncertainty $\boldsymbol{\Delta} = \{\delta I \mid \delta \in \mathbb{C}\}$

In this case, $\mu_{\boldsymbol{\Delta}}(M) = \rho(M)$ holds. Here, $\rho(M)$ denotes the spectral radius of matrix M, that is, the maximum absolute value of all eigenvalues of M.

Proof. First, $\det(I - M\delta) = \det(\delta^{-1}I - M)\det(\delta I) = 0$ holds. So any nonzero δ^{-1} satisfying this equation is an eigenvalue of M. Then, it is easy to see that the reciprocal of the minimum size of uncertainty δ satisfying this equation is the spectral radius $\rho(M)$ of matrix M, that is,

$$\mu_{\boldsymbol{\Delta}}(M) = \frac{1}{\min(|\delta| : \det(I - M\delta) = 0)}$$

Robust Stability and Robust Performance

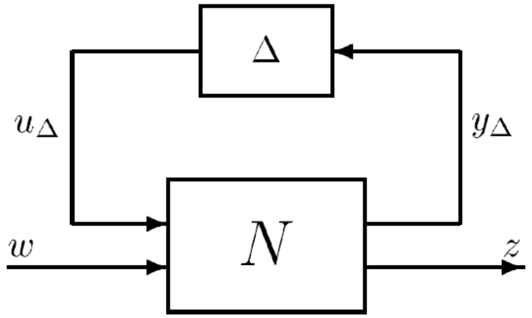


Figure 8.2: $N\Delta$-structure for robust performance analysis

$$
\begin{aligned}
\text{NS} \quad &\Leftrightarrow \quad N \text{ (internally) stable} \\
\text{NP} \quad &\Leftrightarrow \quad \bar{\sigma}(N_{22}) = \mu_{\Delta_P} < 1, \ \forall\omega, \ \text{and NS} \\
\text{RS} \quad &\Leftrightarrow \quad \mu_{\Delta}(N_{11}) < 1, \ \forall\omega, \ \text{and NS} \\
\text{RP} \quad &\Leftrightarrow \quad \mu_{\tilde{\Delta}}(N) < 1, \ \forall\omega, \ \tilde{\Delta} = \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_P \end{bmatrix}, \ \text{and NS}
\end{aligned}
$$

μ synthesis and DK-iteration

- $\max \rho(QN) \leqq \mu(N) \leqq \inf \bar{\sigma}(DND^{-1})$
- $\min_{K} (\min_{D \in \mathcal{D}} \|DN(K)D^{-1}\|_{\infty})$

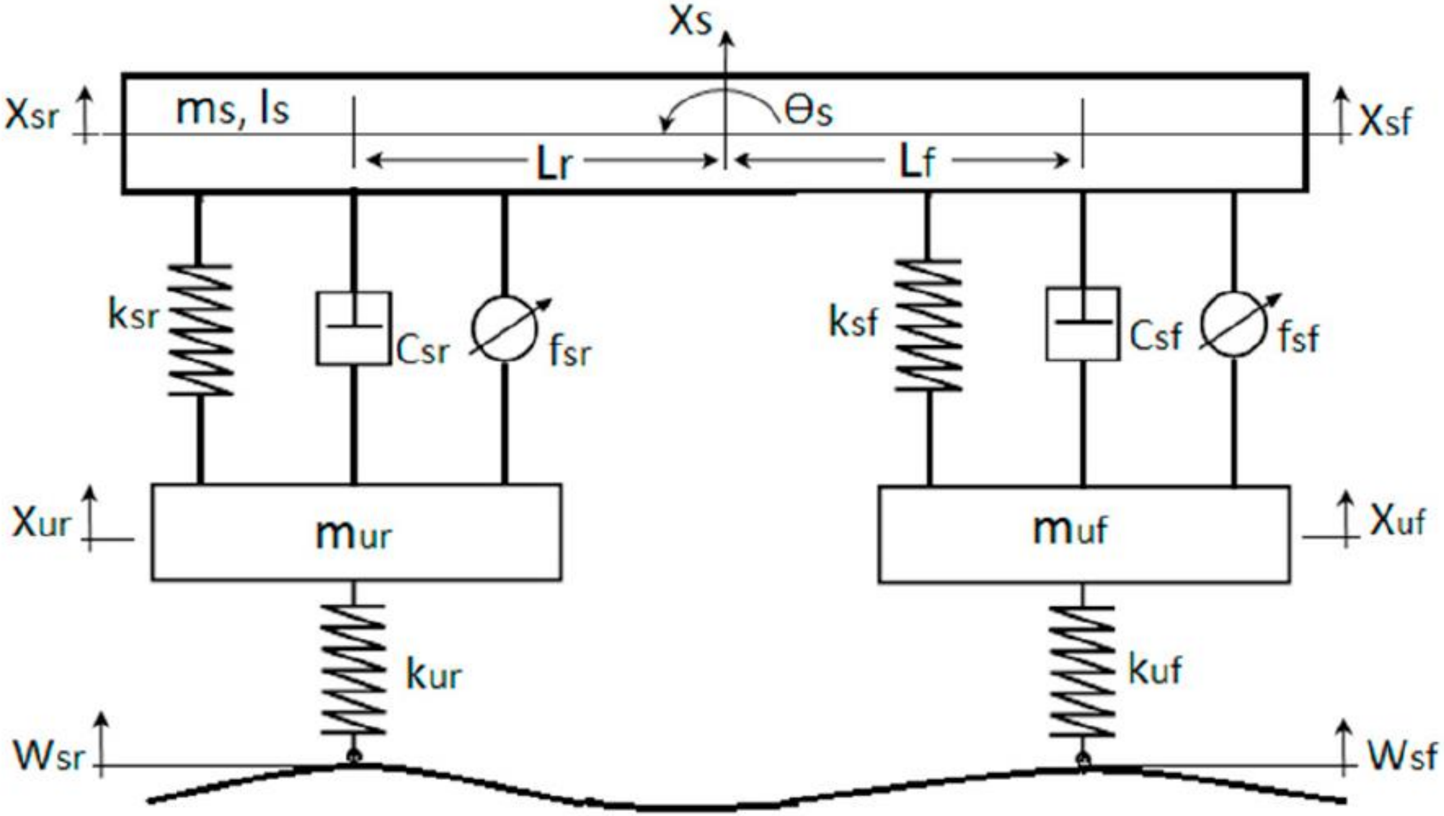
DK-iteration

1. **K-step.** Synthesize an $\mathcal{H}_\infty$ controller for the scaled problem, $\min_K \|DN(K)D^{-1}\|_\infty$ with fixed $D(s)$.
2. **D-step.** Find $D(j\omega)$ to minimize at each frequency $\bar{\sigma}(DND^{-1}(j\omega))$ with fixed N.
3. Fit the magnitude of each element of $D(j\omega)$ to a stable and minimum phase transfer function $D(s)$ and go to Step 1.

Problem Statement

Ride Comfort and Suspension Deflection

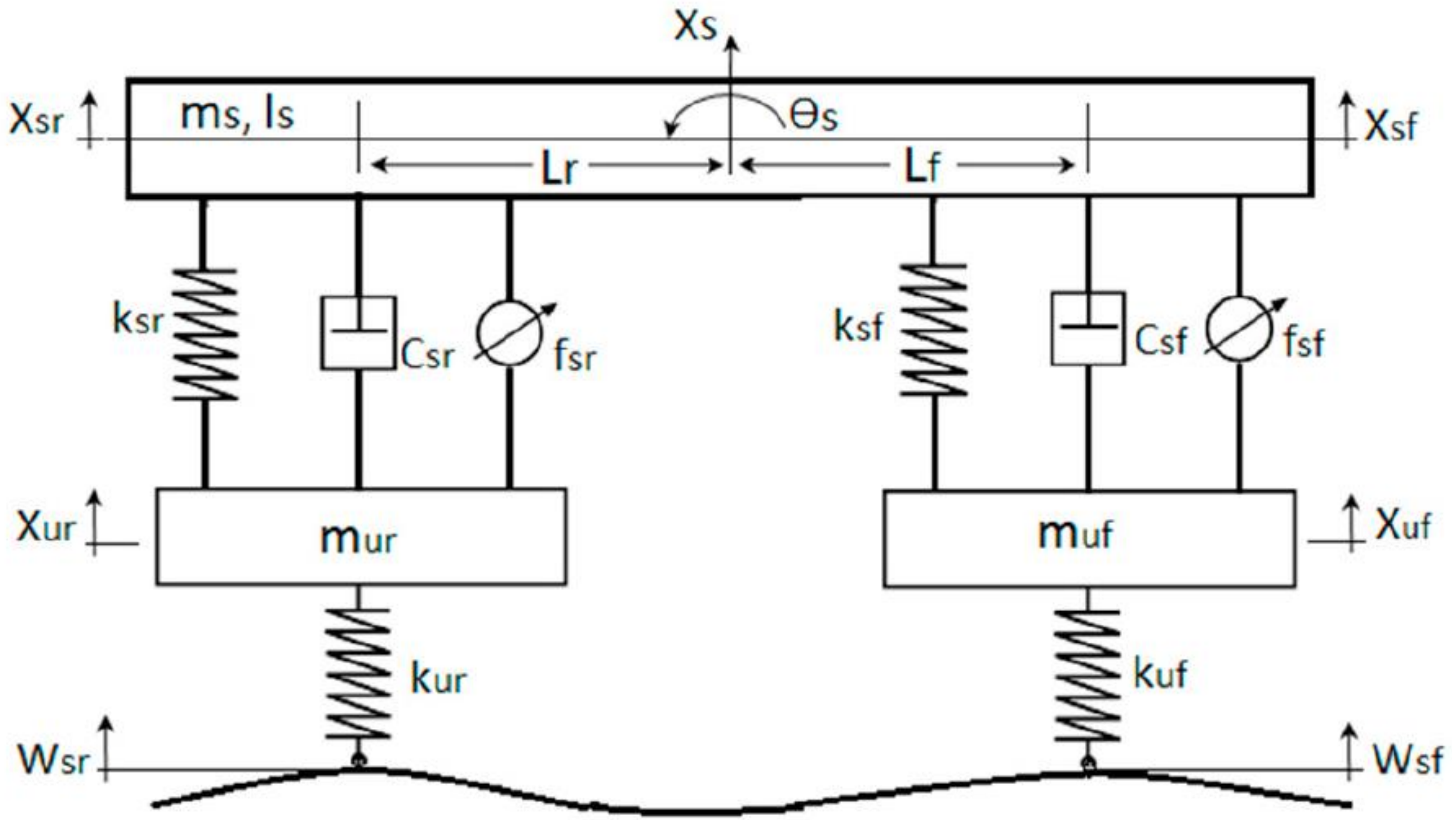
Quantification



A Half-Car Active Suspension System

Quantification

- Ride Comfort and Suspension Deflection?

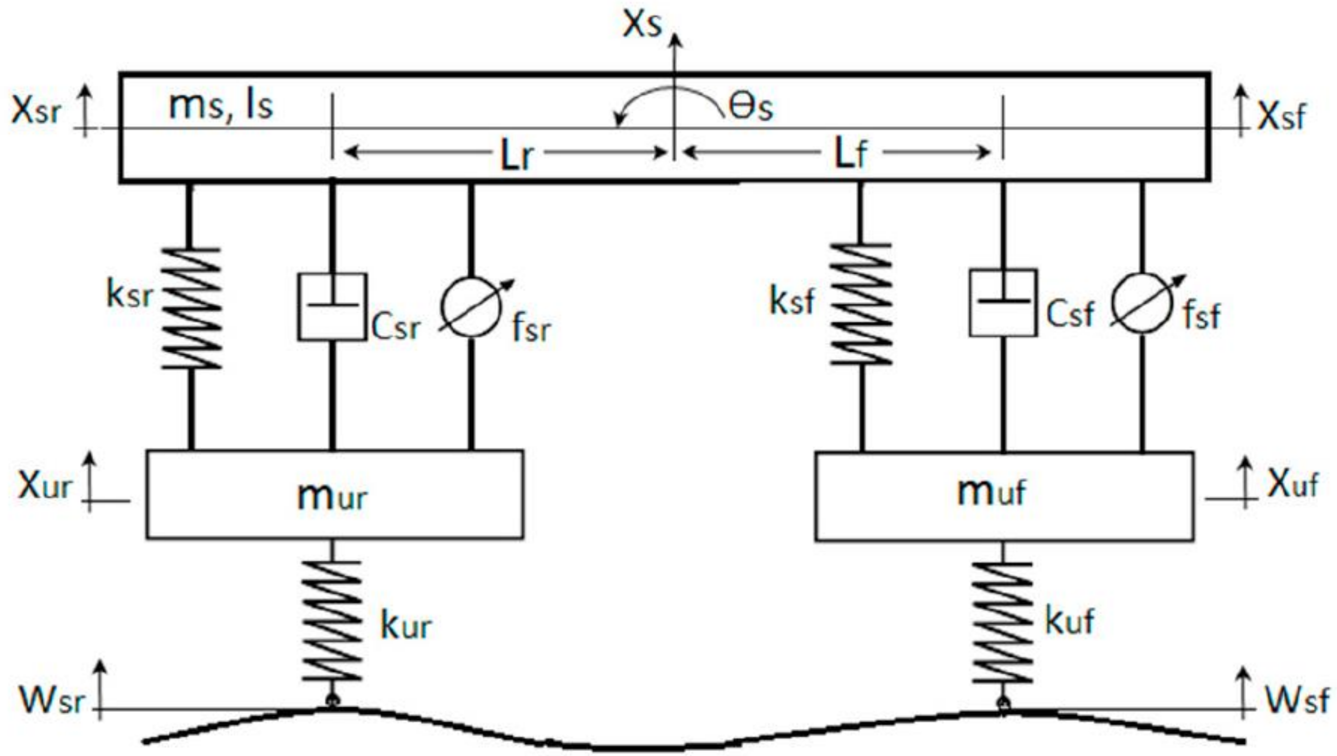


A Half-Car Active Suspension System

Quantification

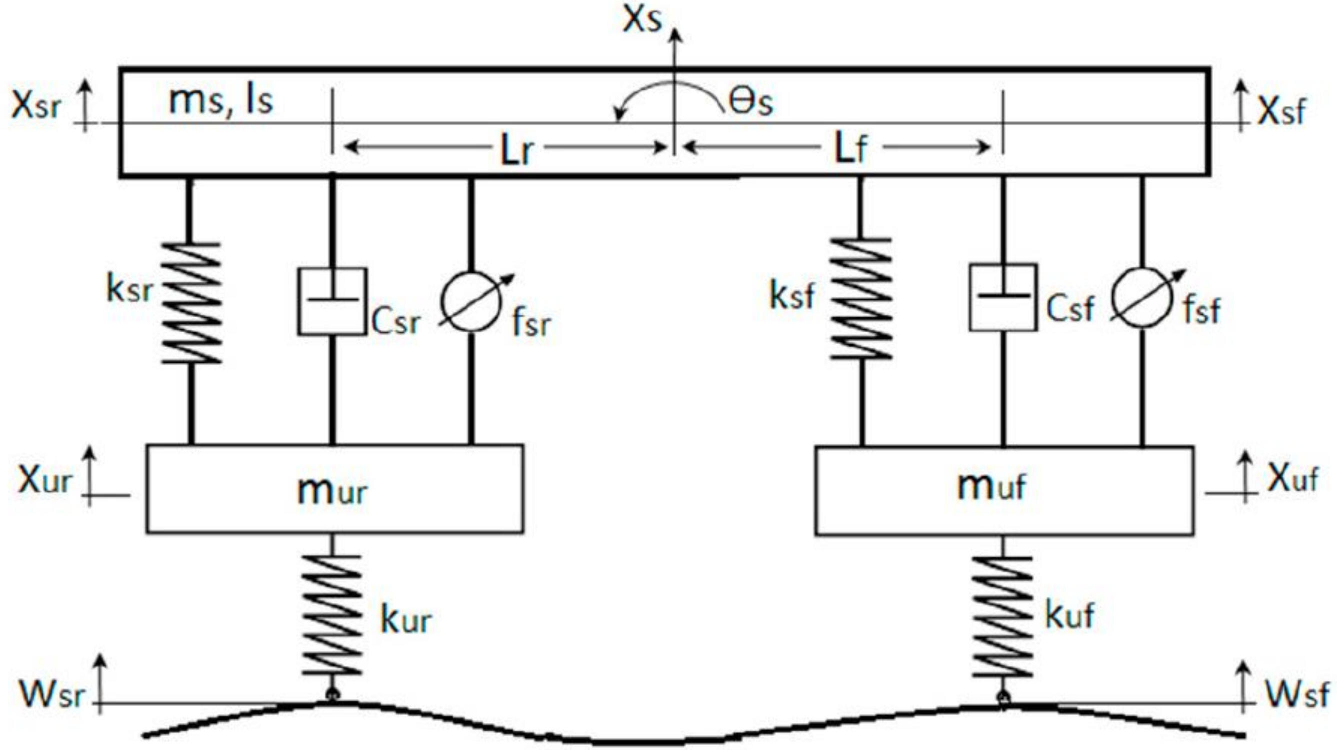
- In general,

  $\ddot{X}_s$ represents ride comfort of a vehicle.



Quantification

- $(\mathrm{X}_{sf} - \mathrm{X}_{uf})$ represents the suspension deflection.
- $(\mathrm{X}_{sr} - \mathrm{X}_{ur})$ represents the suspension deflection.

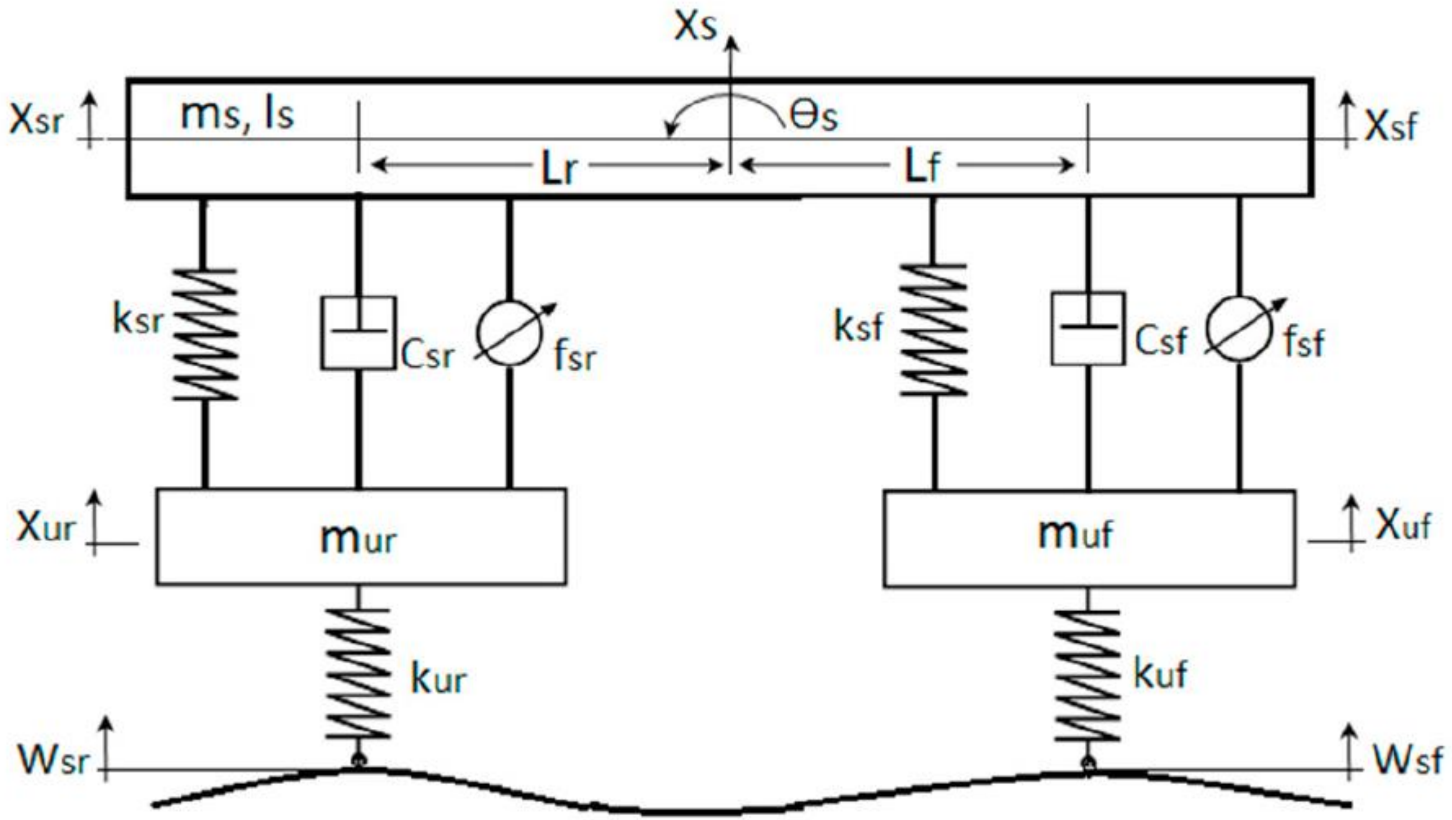


System Structure

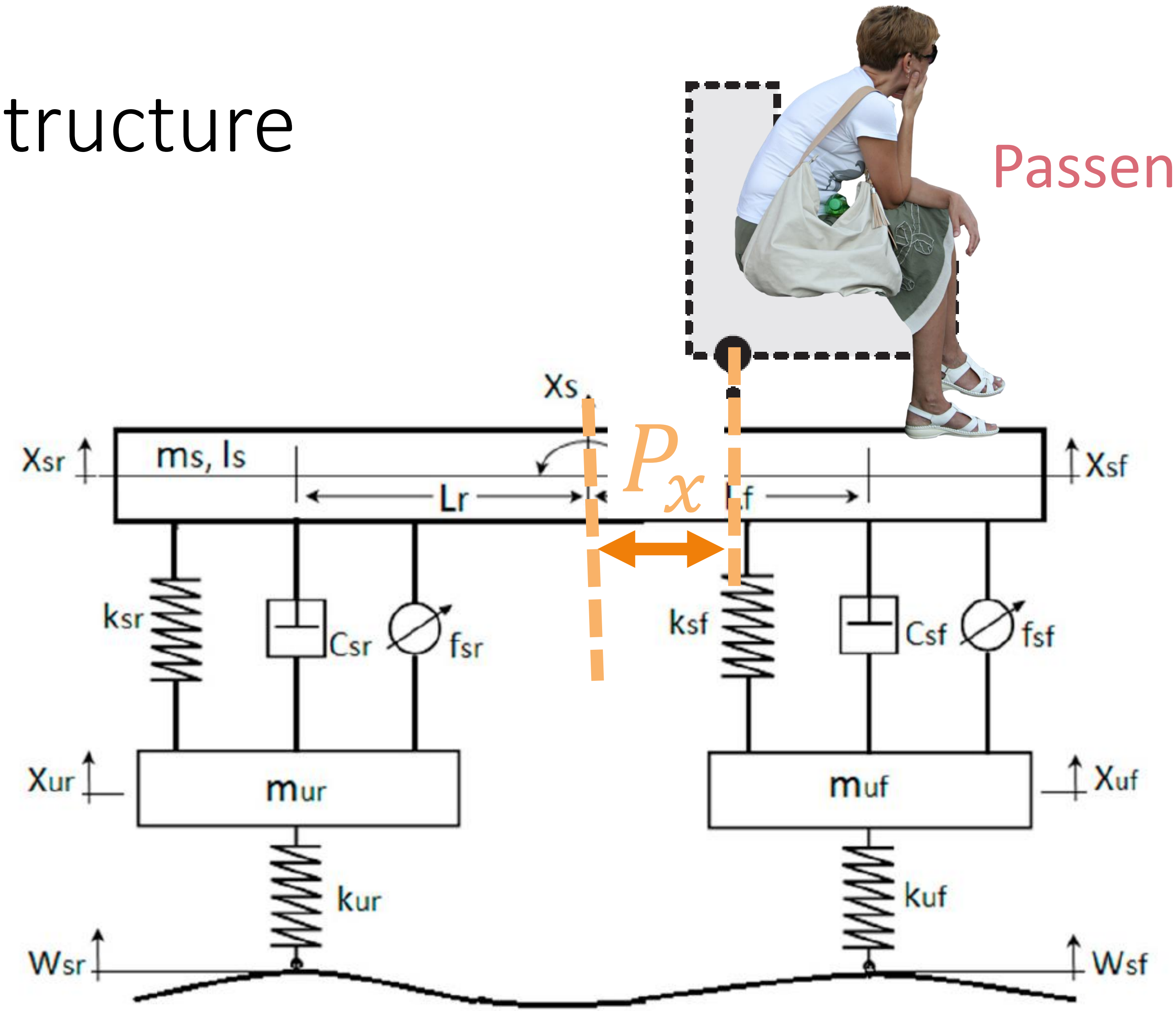
Suspension System + Passenger Model + Hydraulic System

Overall Structure

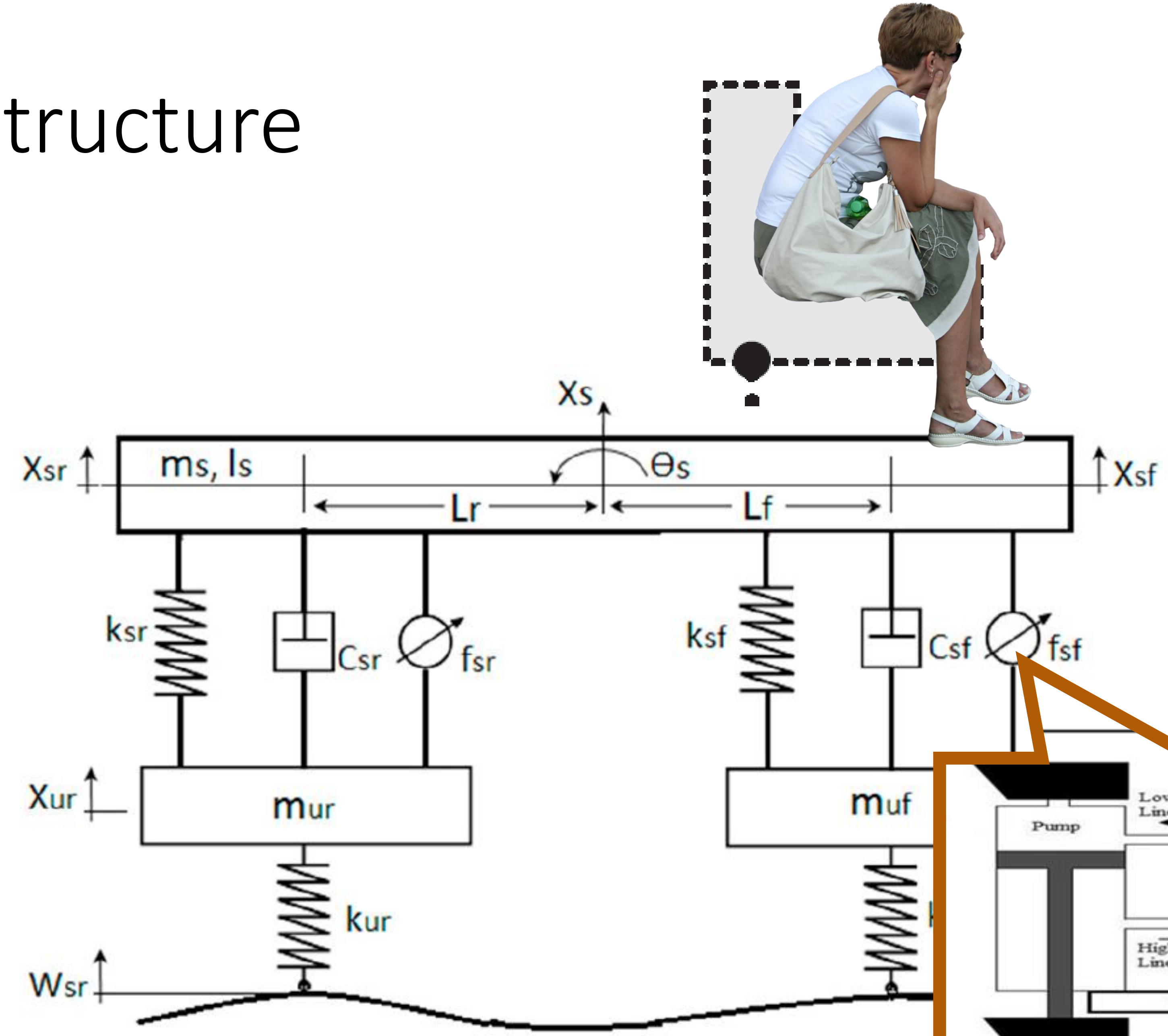
A Half-Car Active Suspension System



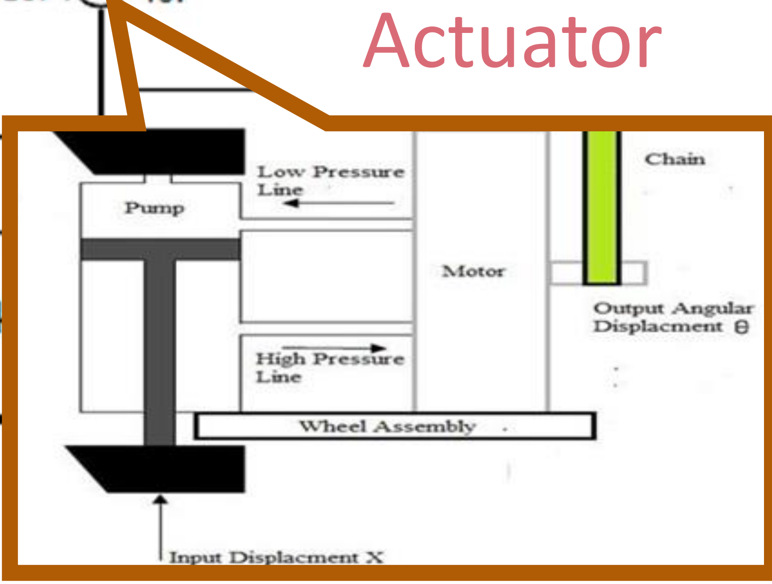
Overall Structure



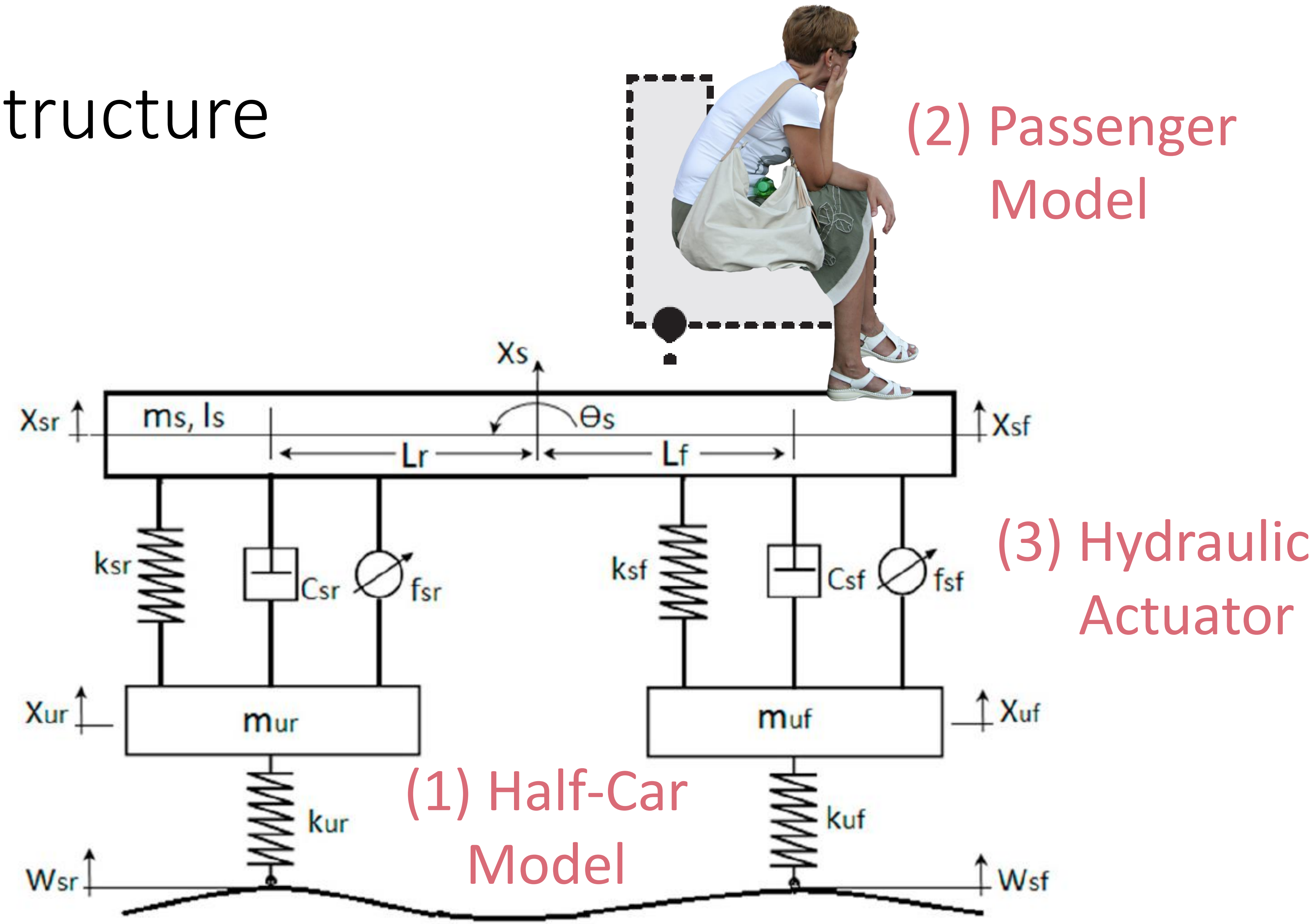
Overall Structure



Hydraulic Actuator



Overall Structure



Half-Car Model

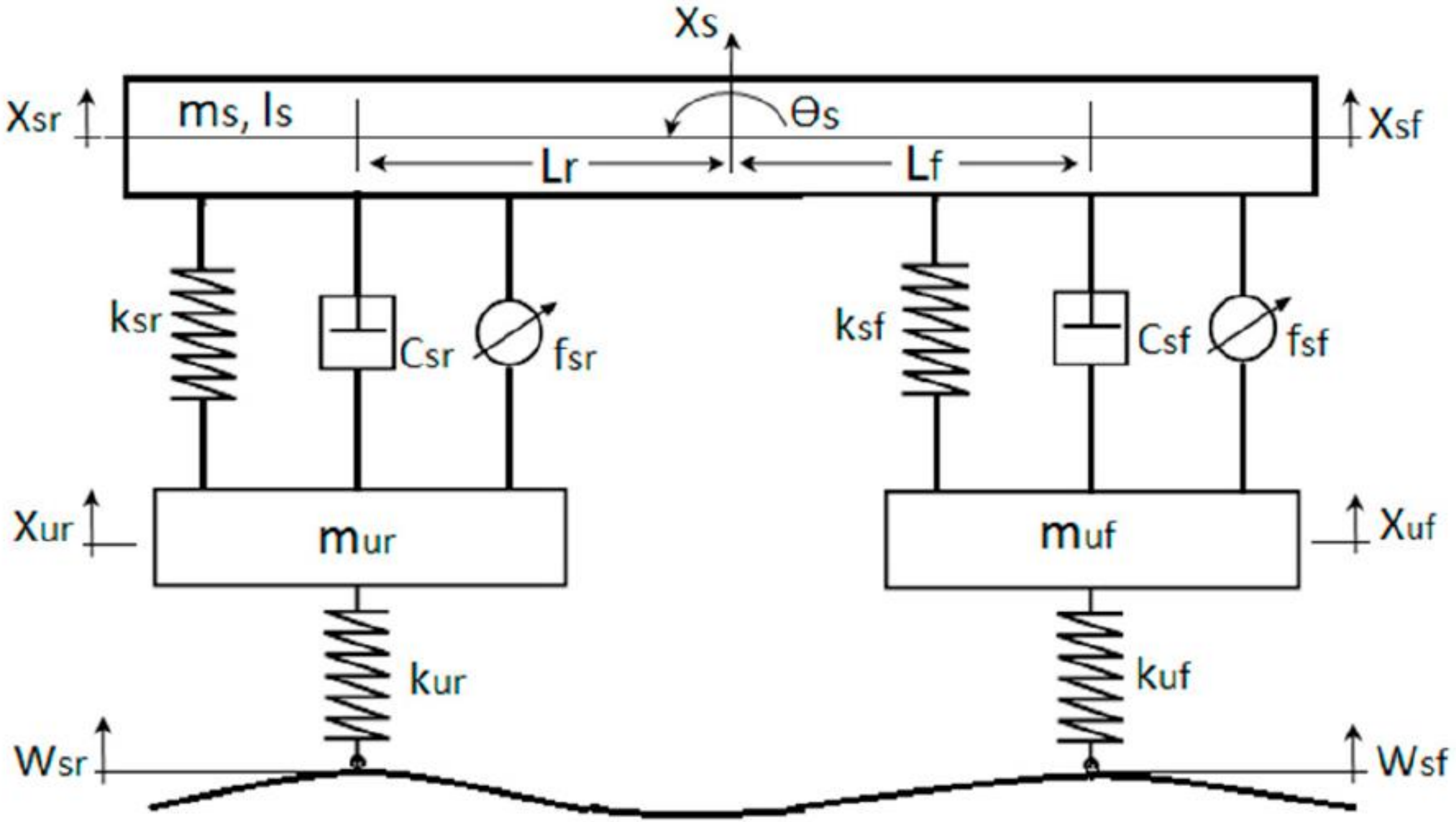


Fig. 1. Half car 4 DOF active suspension model

Half-Car Model

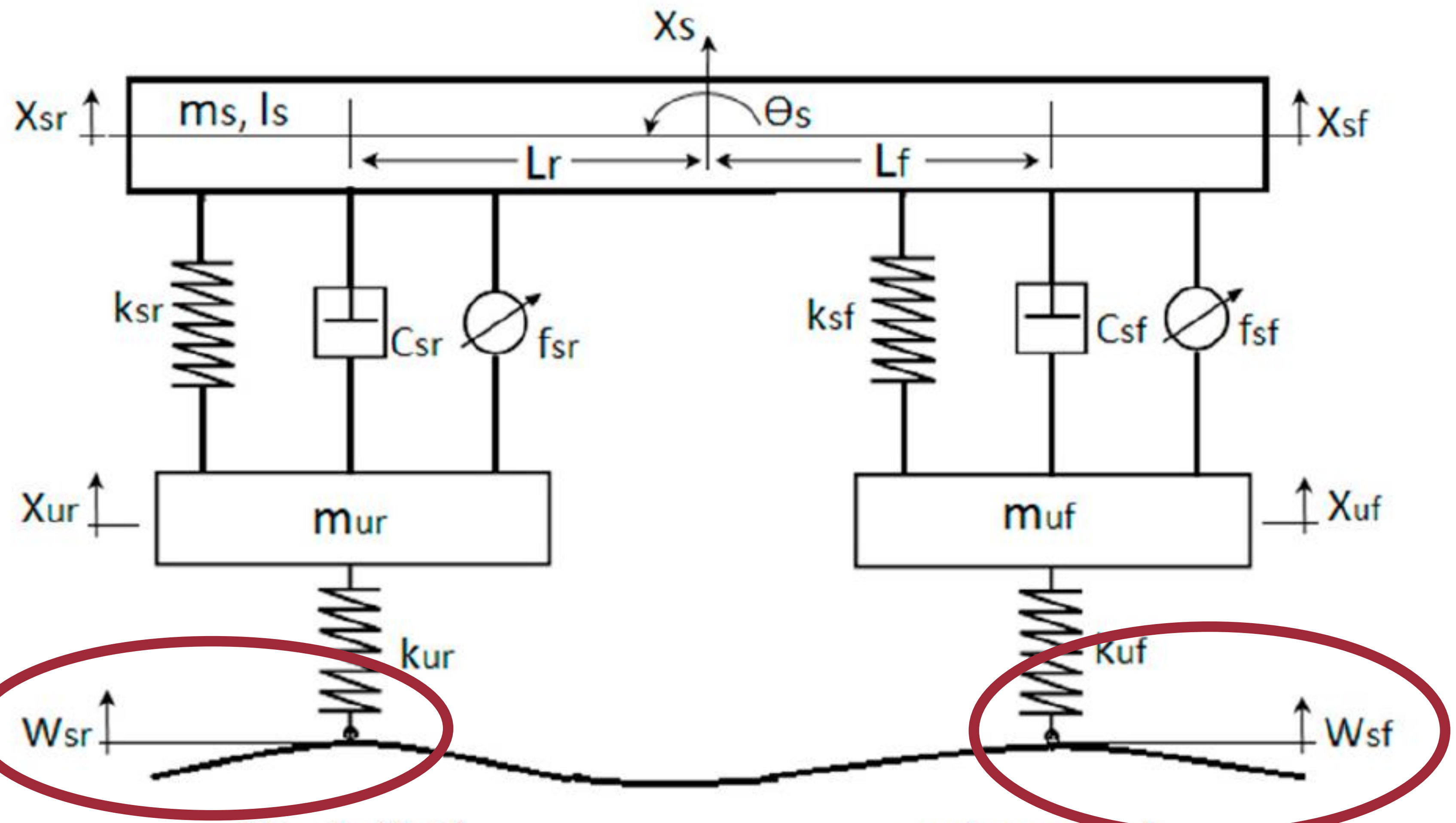


Fig. 1. Half Uneven Road ension model

Half-Car Model

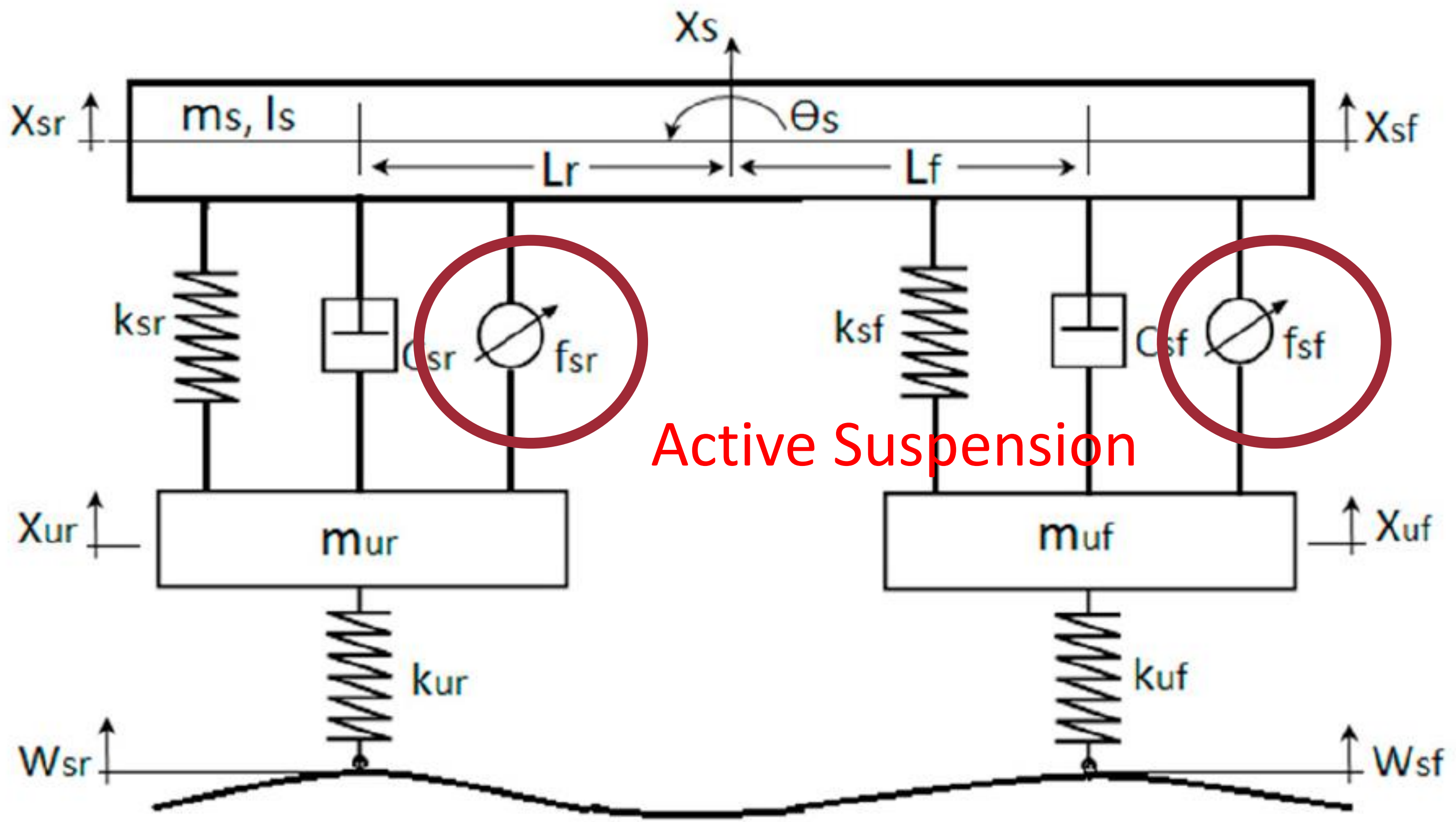


Fig. 1. Half car 4 DOF active suspension model

Half-Car Model

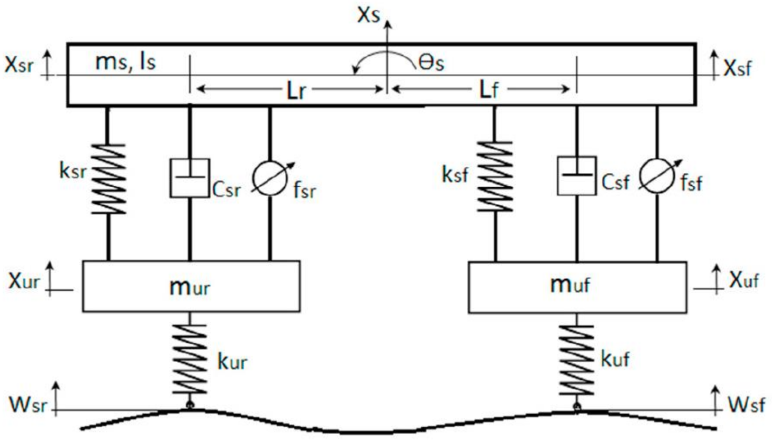


Fig. 1. Half car 4 DOF active suspension model

- Equations of motion

$$m_s \ddot{X}_s + c_{sf}(\dot{X}_{sf} - \dot{X}_{uf}) + k_{sf}(X_{sf} - X_{uf}) + c_{sr}(\dot{X}_{sr} - \dot{X}_{ur}) + k_{sr}(X_{sr} - X_{ur}) - f_{sf} - f_{sr} = 0 \qquad (1)$$

$$I_s \ddot{\theta}_s + L_f\left[c_{sf}(\dot{X}_{sf} - \dot{X}_{uf}) + k_{sf}(X_{sf} - X_{uf}) - f_{sf}\right] - L_r\left[c_{sr}(\dot{X}_{sr} - \dot{X}_{ur}) + k_{sr}(X_{sr} - X_{ur}) - f_{sr}\right] = 0 \quad (2)$$

$$m_{uf} X_{uf} - c_{sf}(X_{sf} - X_{uf}) - k_{sf}(X_{sf} - X_{uf}) + k_{uf}(X_{uf} - w_{sf}) + f_{sf} = 0 \qquad (3)$$

$$m_{ur} \ddot{X}_{ur} - c_{sr}(\dot{X}_{sr} - \dot{X}_{ur}) - k_{sr}(X_{sr} - X_{ur}) + k_{ur}(X_{ur} - w_{sr}) + f_{sr} = 0 \qquad (4)$$

- Constraints

$$X_s = (L_f X_{sr} + L_r X_{sf}) / L$$
$$\theta_s = (X_{sf} - X_{sr}) / L$$

Half-Car Model

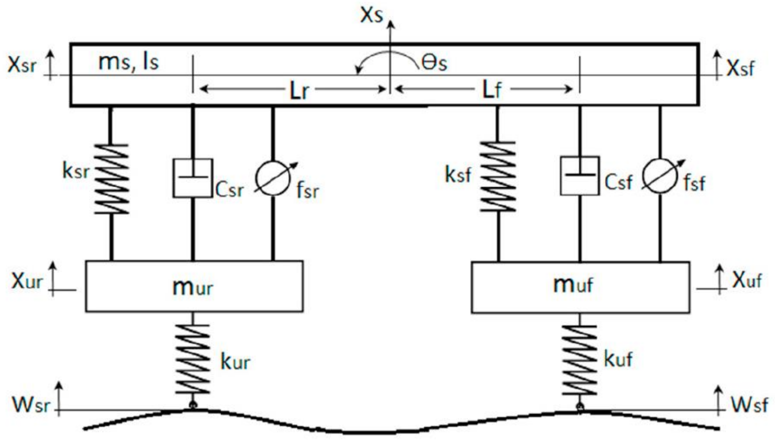


Fig. 1. Half car 4 DOF active suspension model

- $\dot{x} = Ax + Bu$
- $y = Cx + Du$

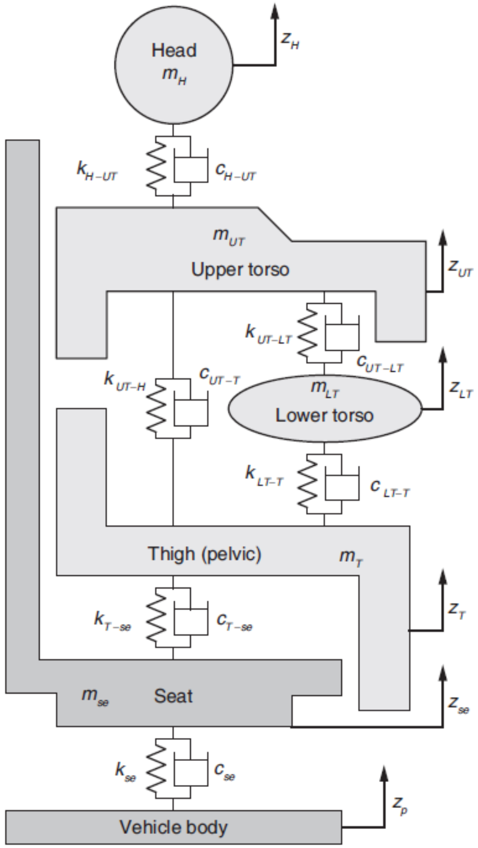
$$x = \left[\dot{x}_{sf}\ \ \dot{x}_{uf}\ \ \dot{x}_{sr}\ \ \dot{x}_{ur}\ \ x_{sf}\ \ x_{uf}\ \ x_{sr}\ \ x_{ur}\right]^T$$

$$u = [\ w_{sf}\ \ w_{sr}\ \ f_{sf}\ \ f_{sr}\ ]^T$$

Passenger Model

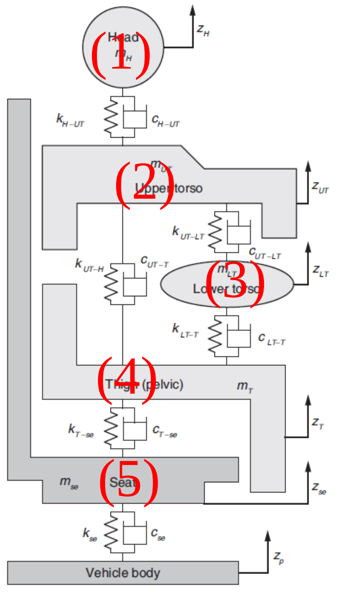
Passenger Model

- Uncertain Biodynamics



Passenger Model

- Equations of motion



$$m_H \ddot{z}_H = -k_{H-UT}\left(z_H - z_{UT}\right) - c_{H-UT}\left(\dot{z}_H - \dot{z}_{UT}\right) \tag{1}$$

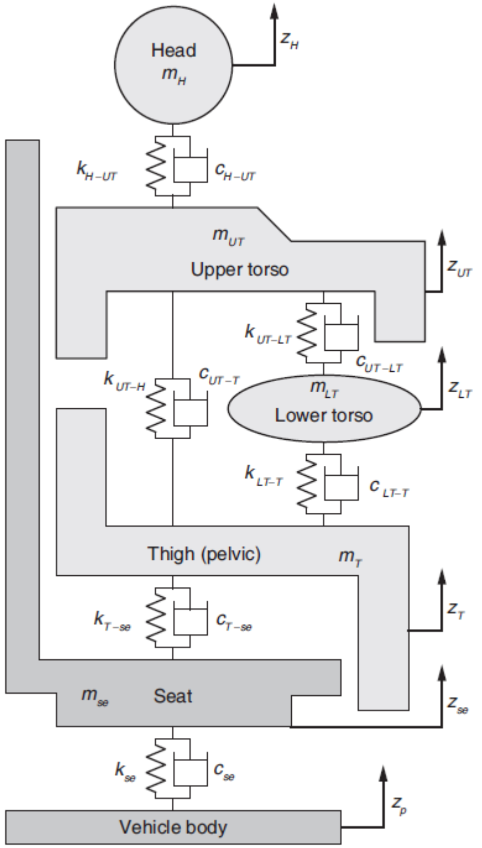
$$\begin{aligned} m_{UT}\ddot{z}_{UT} &= k_{H-UT}\left(z_H - z_{UT}\right) - k_{UT-LT}\left(z_{UT} - z_{LT}\right) - k_{UT-T}\left(z_{UT} - z_T\right) \\ &\quad + c_{H-UT}\left(\dot{z}_H - \dot{z}_{UT}\right) - c_{UT-LT}\left(\dot{z}_{UT} - \dot{z}_{LT}\right) - c_{UT-T}\left(\dot{z}_{UT} - \dot{z}_T\right) \end{aligned} \tag{2}$$

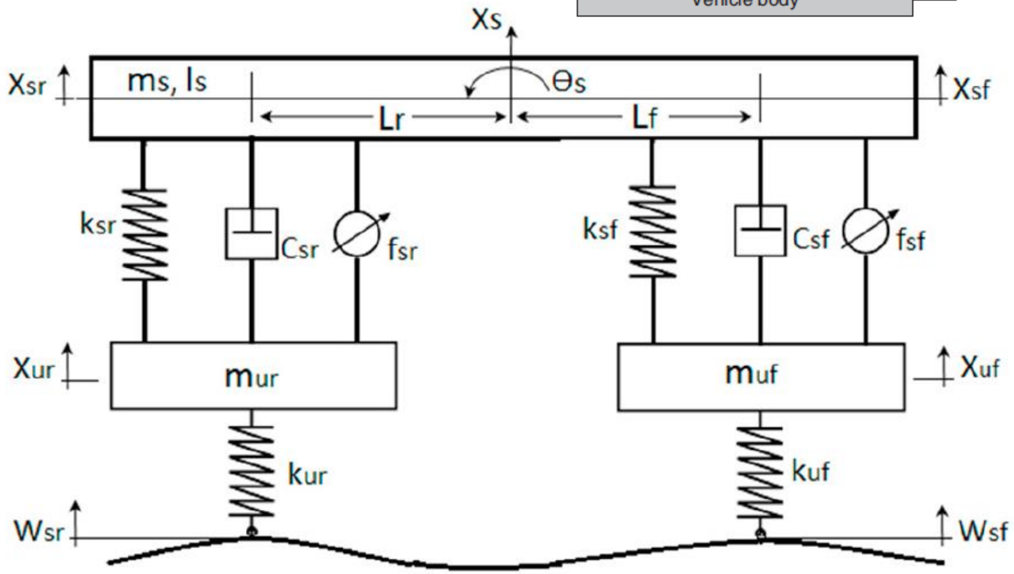
$$\begin{aligned} m_{LT}\ddot{z}_{LT} &= k_{UT-LT}\left(z_{UT} - z_{LT}\right) - k_{LT-T}\left(z_{LT} - z_T\right) + c_{UT-LT}\left(\dot{z}_{UT} - \dot{z}_{LT}\right) \\ &\quad - c_{LT-T}\left(\dot{z}_{LT} - \dot{z}_T\right) \end{aligned} \tag{3}$$

$$\begin{aligned} m_T\ddot{z}_T &= k_{UT-T}\left(z_{UT} - z_T\right) + k_{LT-T}\left(z_{LT} - z_T\right) - k_{T-se}\left(z_T - z_{se}\right) \\ &\quad + c_{UT-T}\left(\dot{z}_{UT} - \dot{z}_T\right) + c_{LT-T}\left(\dot{z}_{LT} - \dot{z}_T\right) - c_{T-se}\left(\dot{z}_T - \dot{z}_{se}\right) \end{aligned} \tag{4}$$

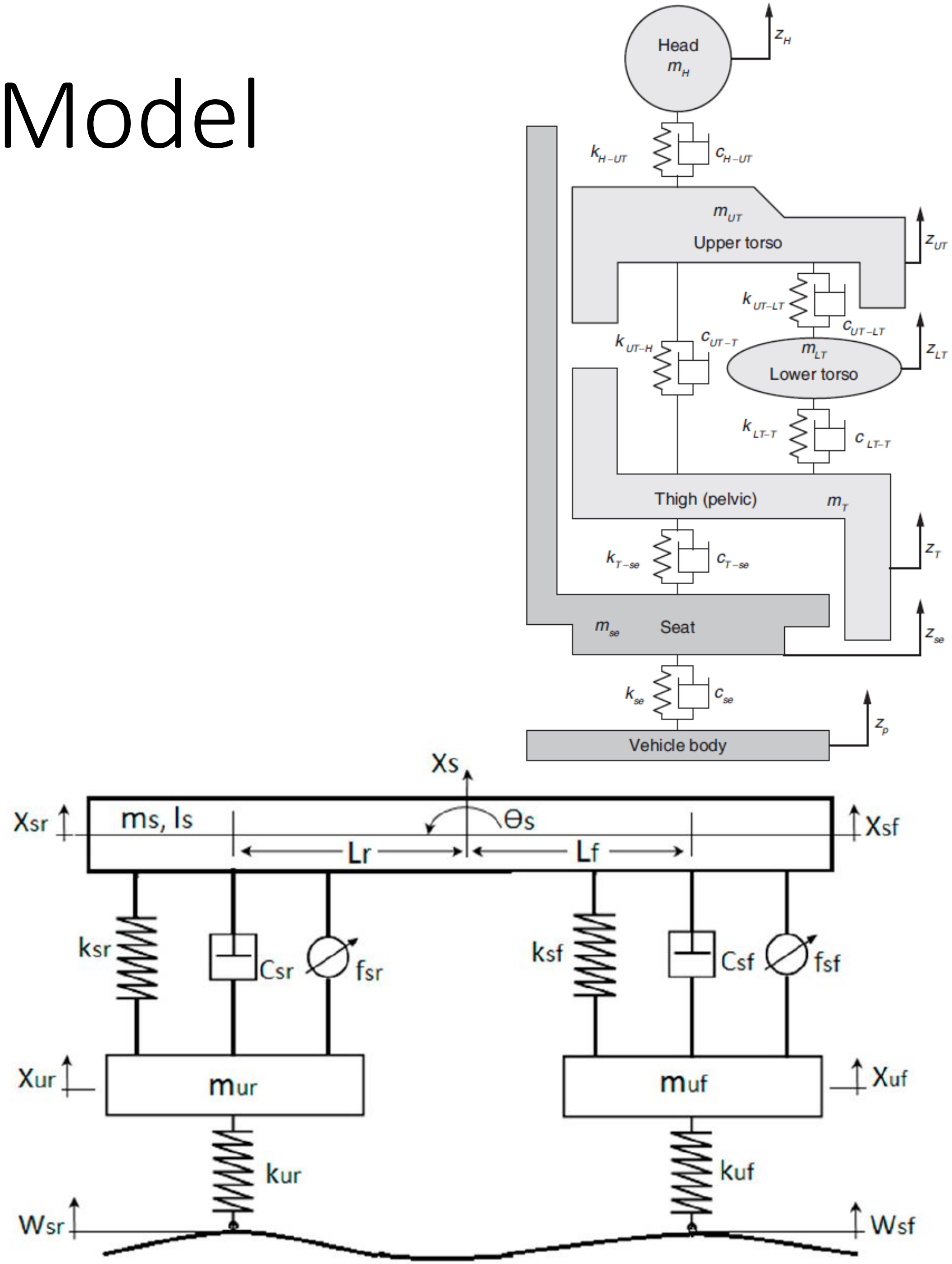
$$\begin{aligned} m_{se}\ddot{z}_{se} &= k_{T-se}\left(z_T - z_{se}\right) - k_{se}\left(z_{se} - z_p\right) + c_{T-se}\left(\dot{z}_T - \dot{z}_{se}\right) \\ &\quad - c_{se}\left(\dot{z}_{se} - \dot{z}_p\right) \end{aligned} \tag{5}$$

Passenger Model





Passenger Model



$$z_p = x_s + P_x\,\theta_s$$

Hydraulic Actuator

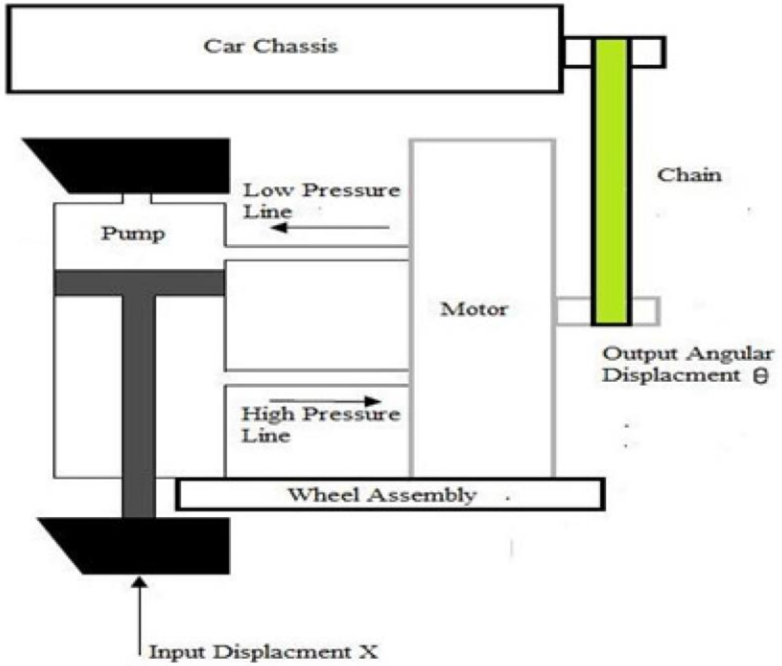


Figure 2: Hydraulic actuator block diagram

Hydraulic Actuator

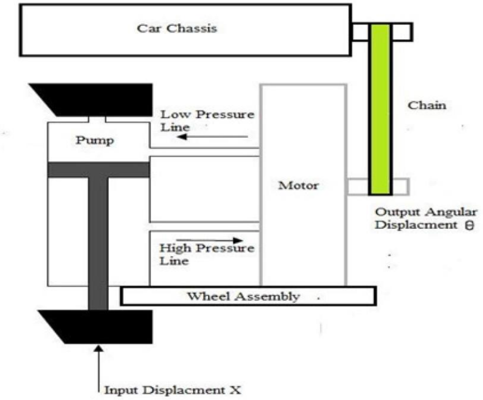
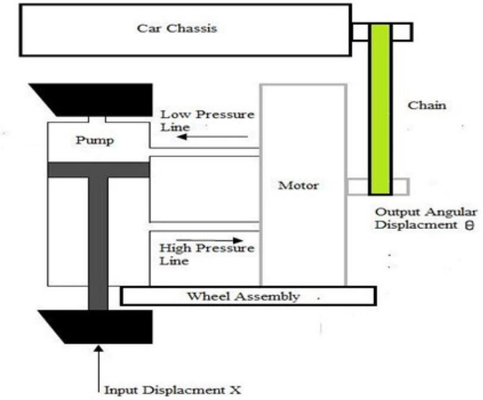


Figure 2: Hydraulic actuator block diagram

- Oil waft from the pump, $q_p = K_p \frac{dx}{dt}$
- Oil glide via the motor, $q_m = K_m \frac{d\theta}{dt}$
- Leakage flow rate, $q_i = K_i P$
- Compressibility flow rate, $q_c = K_c \frac{dp}{dt}$

Hydraulic Actuator



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- Oil glide via the motor, $q_m = K_m \frac{d\theta}{dt}$
- Leakage flow rate, $q_i = K_i P$
- Compressibility flow rate, $q_c = K_c \frac{dp}{dt}$

$$q_p = q_m + q_i + q_c \quad \Rightarrow \quad K_p \frac{dx}{dt} = K_m \frac{d\theta}{dt} + K_i P + K_c \frac{dP}{dt}$$

Hydraulic Actuator

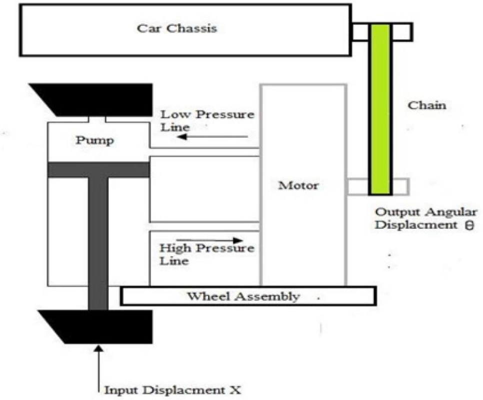


Figure 2: Hydraulic actuator block diagram

- Assumptions:
  - Km = Kt = Kc
  - Tm = Tl
  - Kc = 0
- Transfer function

$$\frac{\theta(s)}{X(s)} = \frac{K_p}{\left[\frac{K_i J}{K_m} s + \frac{K_m^2 + K_i B}{K_m}\right]}$$

Hydraulic Actuator

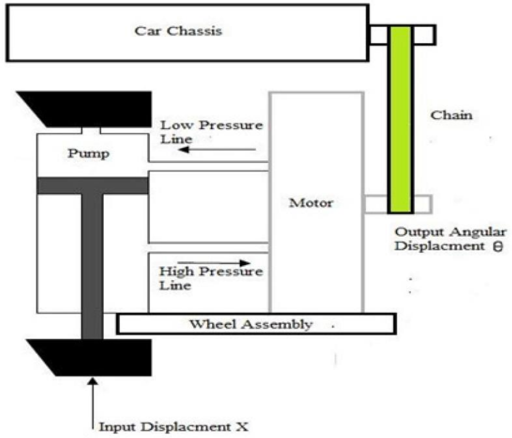
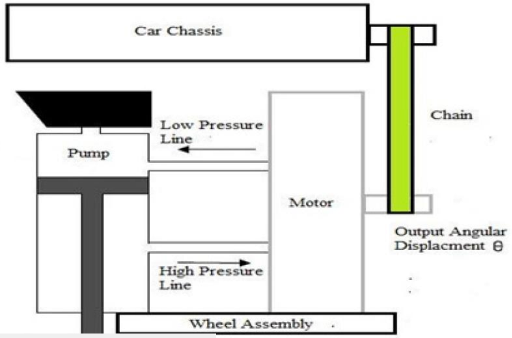


Figure 2: Hydraulic actuator block diagram

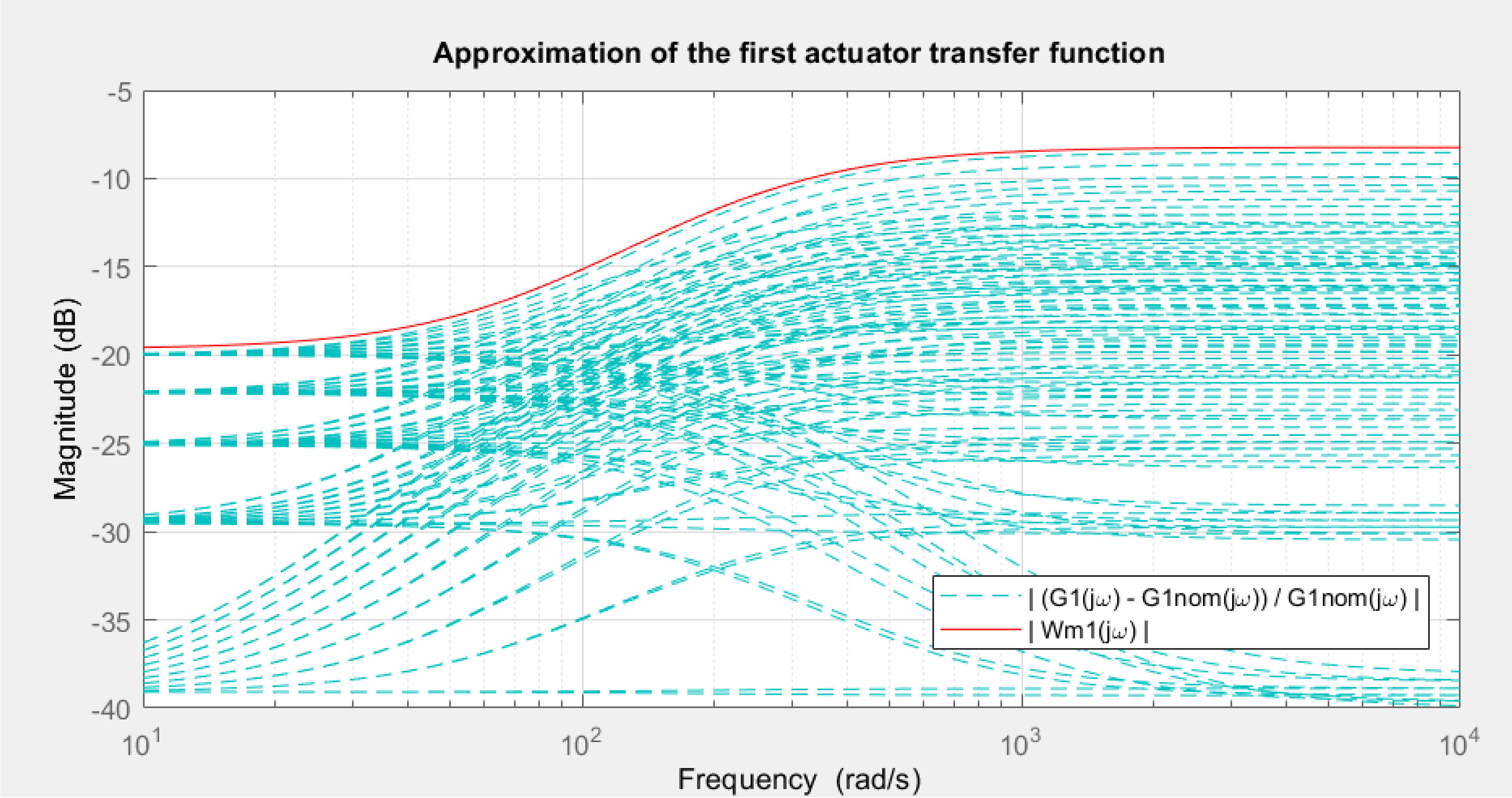
- nominal $G_1(s) = \frac{1.08}{0.005s+1}$
- purturbed $\widetilde{G_1(s)} = \frac{K_1}{T_1 s+1}$ with multiplicative uncertainty
  - $\widetilde{G_1} = G_1\,(1 + W_{m1}\Delta_1)$, where $\|\Delta_1\|_\infty < 1$.
  - K_1: uncertainty of 10%
  - T_1: uncertainty of 20%

Hydraulic Actuator



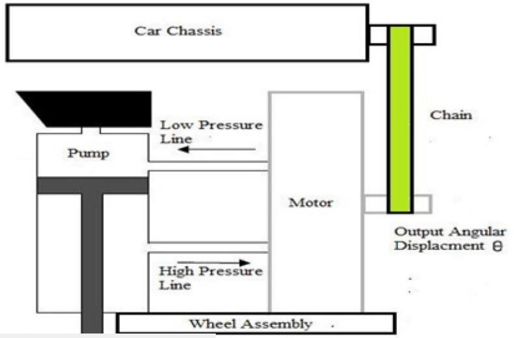
nt X

actuator block diagram

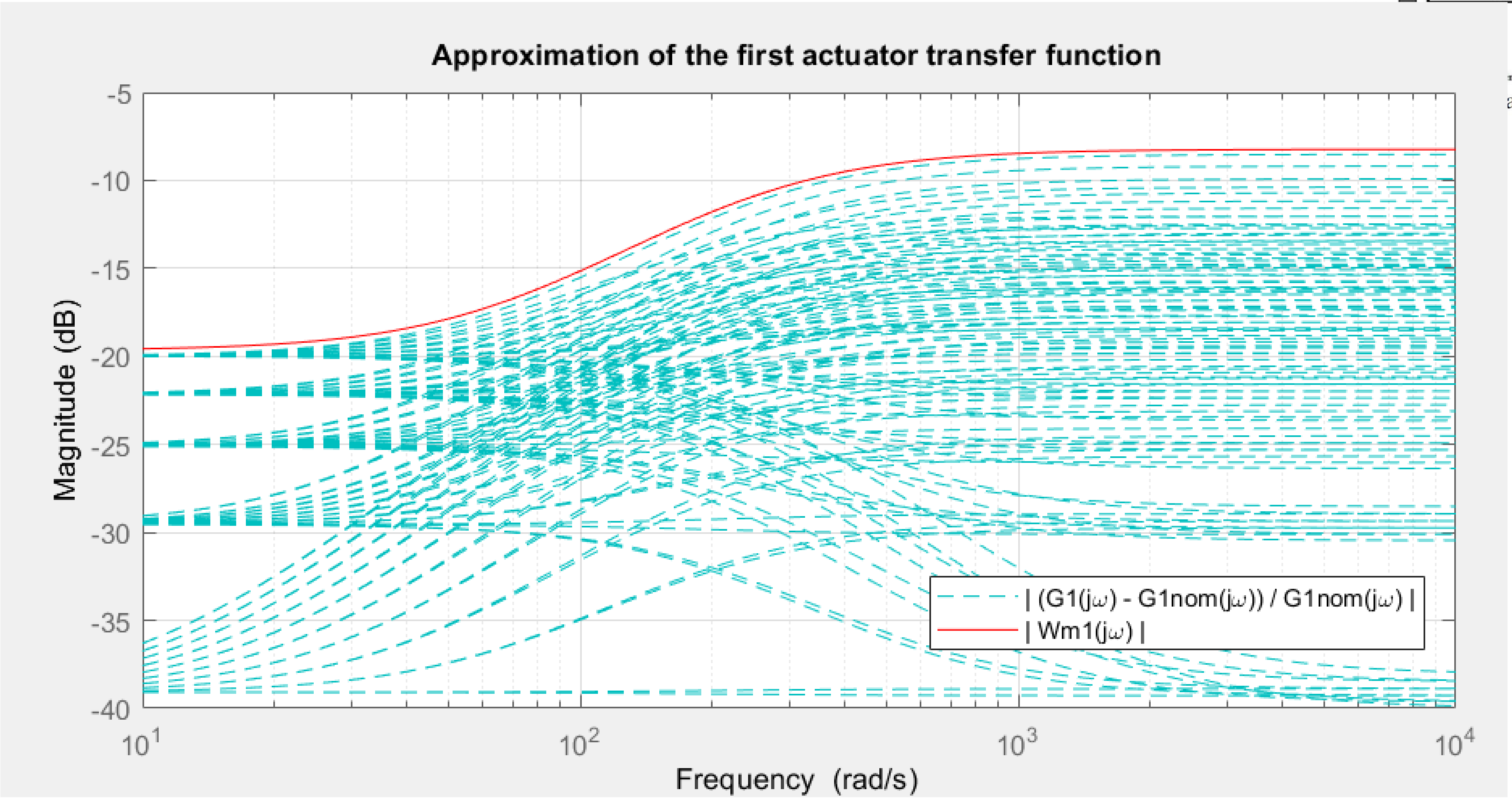


Hydraulic Actuator

$$W_{m1} = \frac{0.3803s + 60.8973}{s + 599.5829}$$

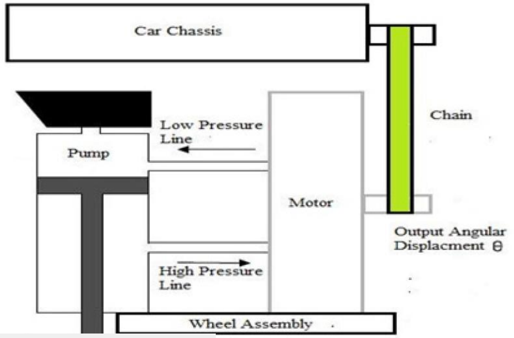


nt X
actuator block diagram



Hydraulic Actuator

$$W_{m1} = \frac{0.3803s + 60.8973}{s + 599.5829}$$



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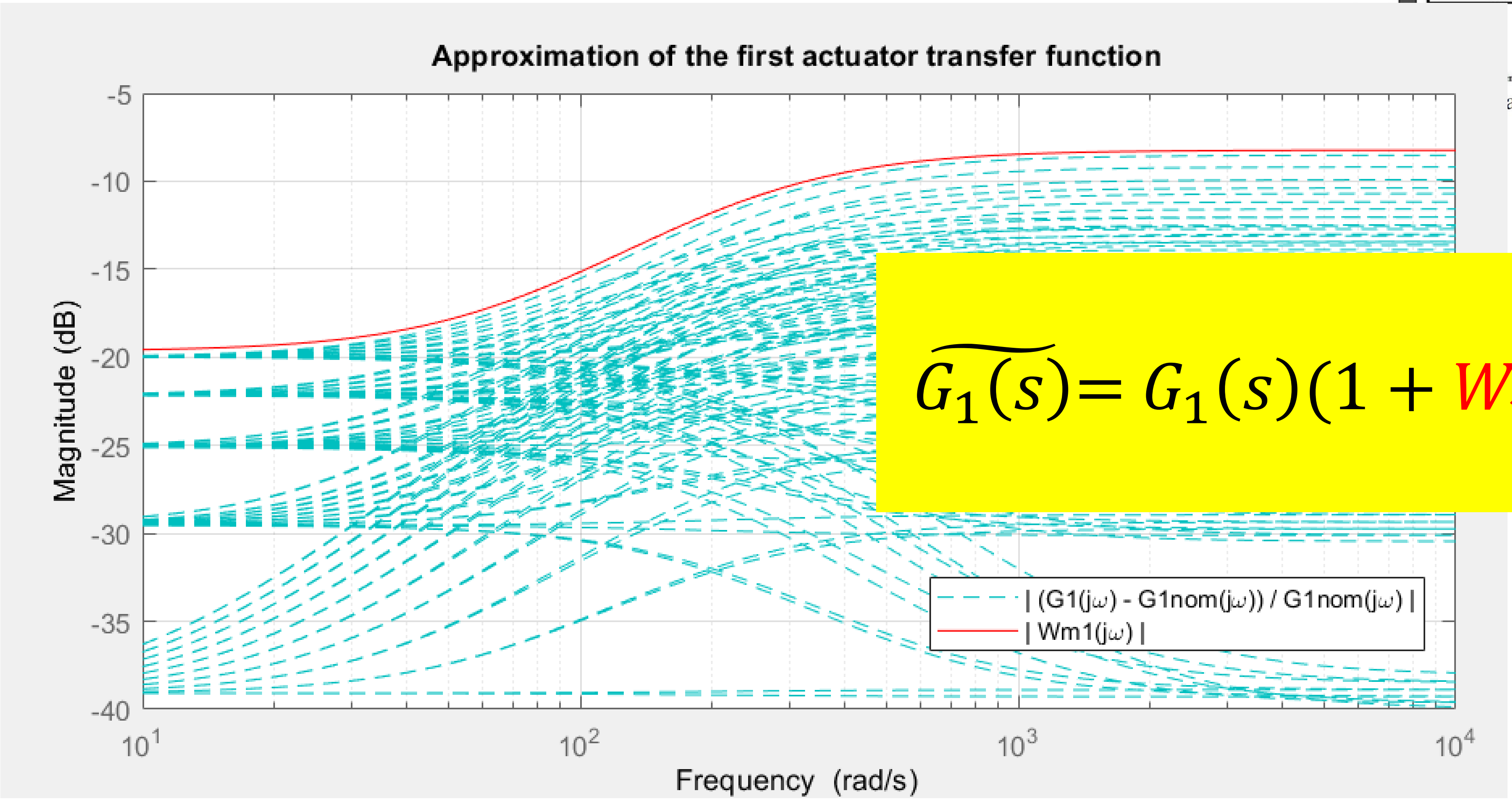
actuator block diagram

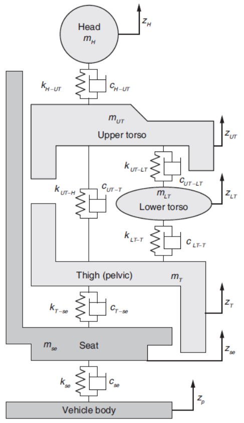


$$\widetilde{G_1(s)} = G_1(s)(1 + W_{m1}\Delta_1)$$

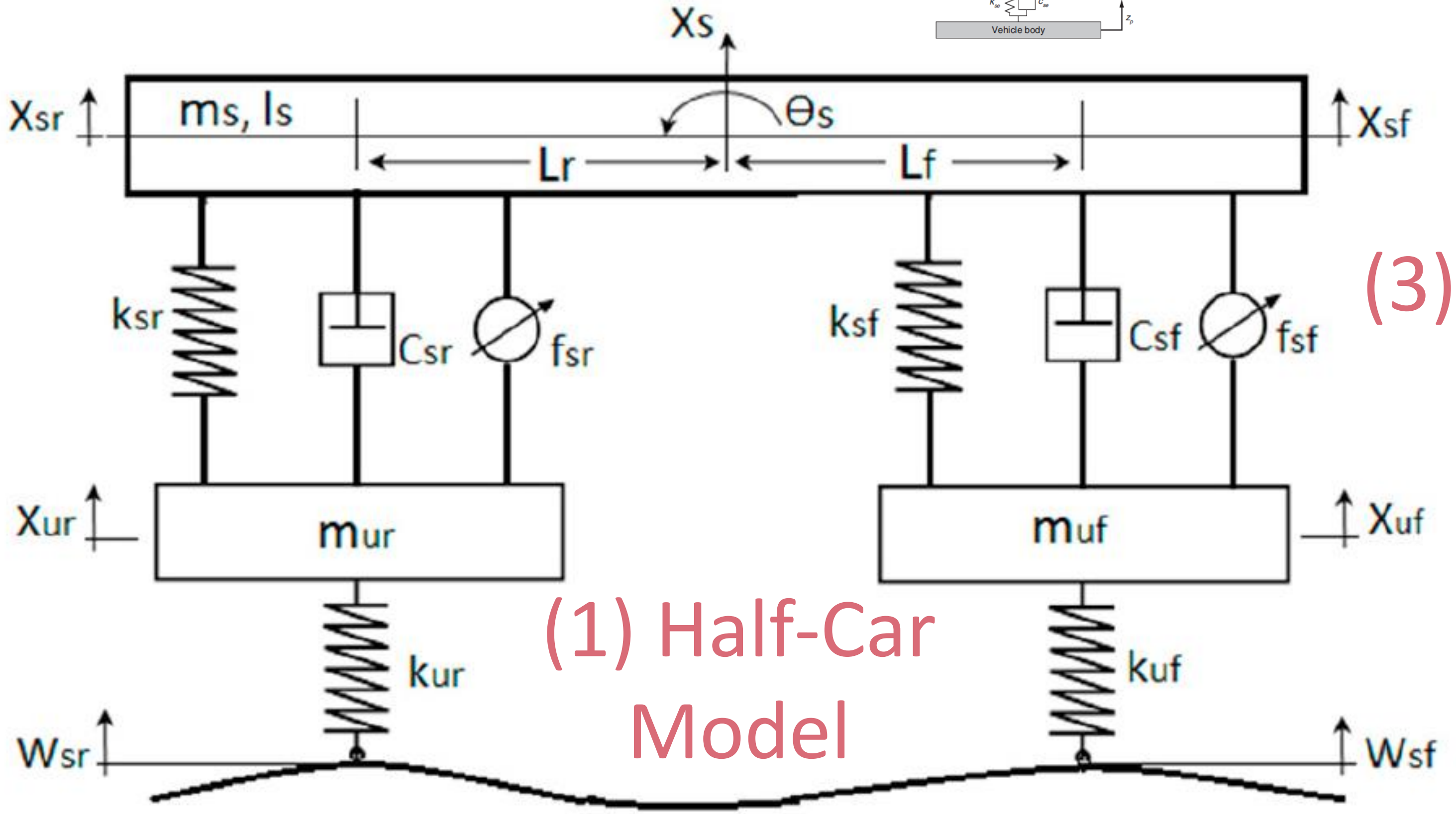
Results

From modeling to μ -synthesis design

Overall Structure



(2) Passenger Model

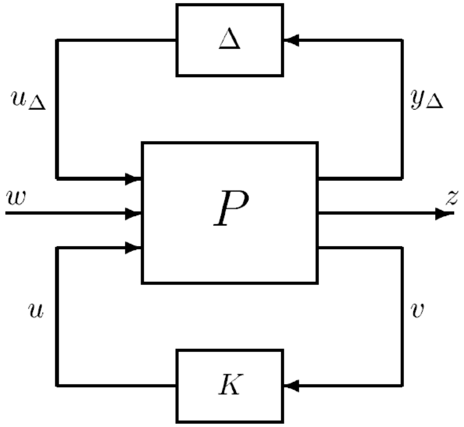


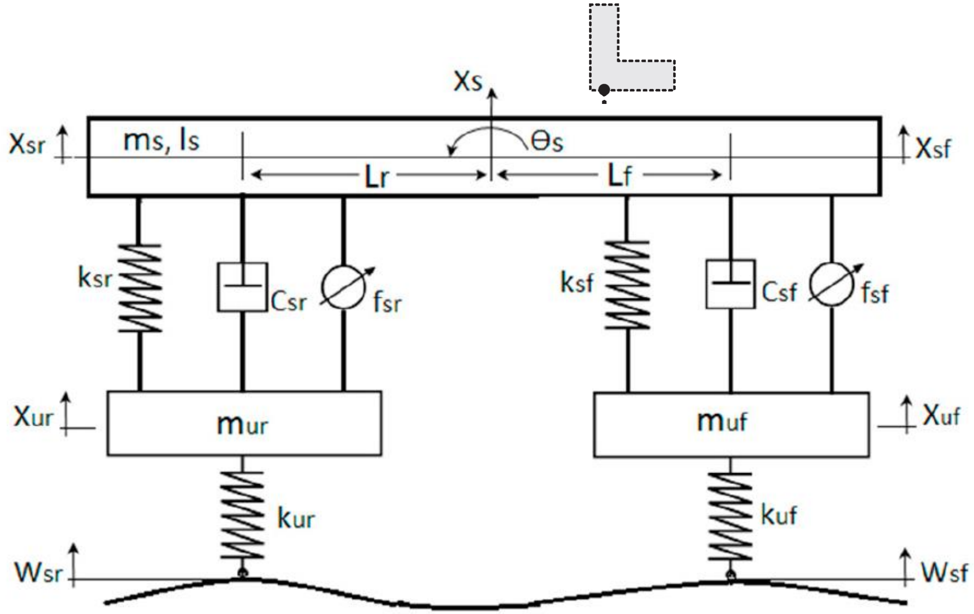
(3) Hydraulic Actuator

(1) Half-Car Model

State-Space

- Plant P
  - $\dot{x} = Ax + Bu$
  - $y = Cx + Du$





State-Space

- $\dot{x} = Ax + Bu$
- $y = Cx + Du$

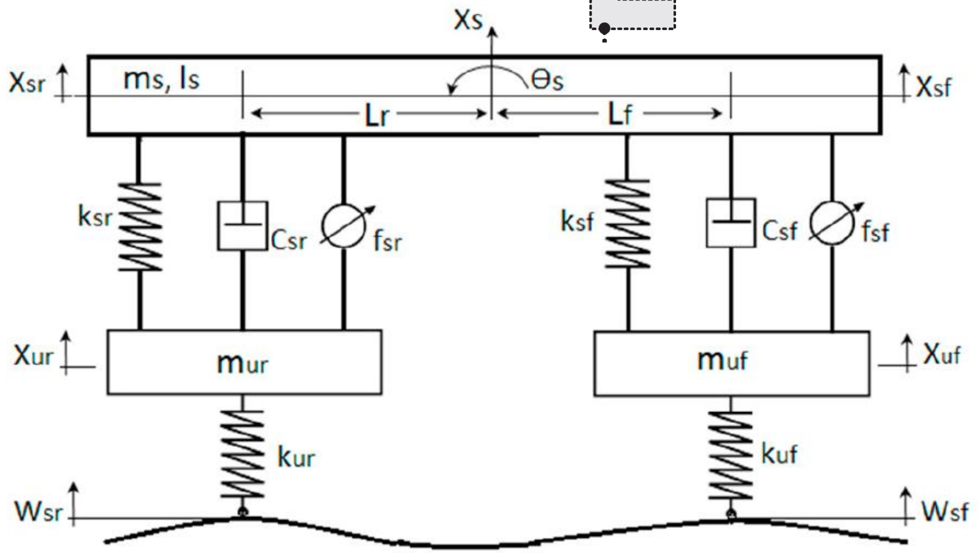
$x =$

hcar.StateName

	1
1	v_sf
2	v_uf
3	v_sr
4	v_ur
5	x_sf
6	x_uf
7	x_sr
8	x_ur
9	Z_H
10	Z_UT
11	Z_LT
12	Z_T
13	Z_se
14	Z_H_dot
15	Z_UT_dot
16	Z_LT_dot
17	Z_T_dot
18	Z_se_dot

$$u^T = [\, w_{sf}, w_{sr}\, , f_{sf}\, , f_{sr}\,]$$

$$y^T = [\, (x_{sf} - x_{uf})\, , (x_{sr} - x_{ur}), \ddot{z}_H]$$



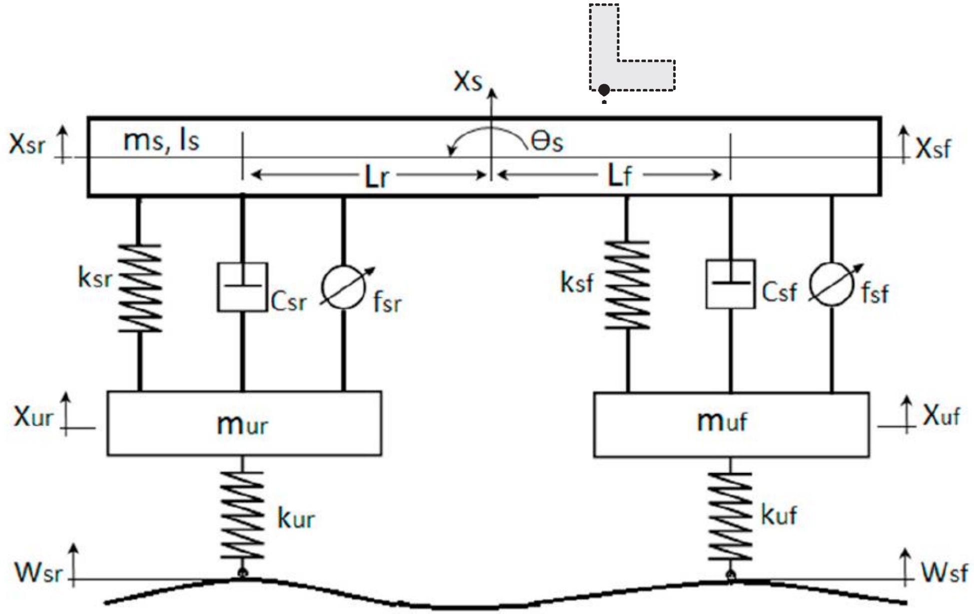
State-Space

```
A = [ -c_sf / m_s - L_f^2 * c_sf / I_s , c_sf / m_s + L_f^2 * c_sf / I_s, - c_sr / m_s + L_
      c_sf / m_uf, - c_sf / m_uf, 0, 0, k_sf / m_uf, - (k_sf + k_uf) / m_uf, 0, 0, 0, 0, 0, 0,
      (-c_sf / m_s + L_f * L_r * c_sf / I_s),  (c_sf / m_s - L_f * L_r * c_sf / I_s), (- c_sr /
      0, 0, c_sr / m_ur, - c_sr / m_ur, 0, 0, k_sr / m_ur, - (k_sr + k_ur) / m_ur, 0, 0, 0, 0,
      1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
      0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
      0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
      0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0; ...
     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0; ...
     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0; ...
     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0; ...
     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1; ...
     0, 0, 0, 0, 0, 0, 0, 0, - K_H_UT / m_H, K_H_UT / m_H, 0, 0, 0, - C_H_UT / m_H, C
     0, 0, 0, 0, 0, 0, 0, 0, K_H_UT / m_UT, - (K_H_UT + K_UT_LT + K_UT_T) / m_UT
     0, 0, 0, 0, 0, 0, 0, 0, 0, K_UT_LT / m_LT, - (K_UT_LT + K_LT_T) / m_LT, K_LT_T
     0, 0, 0, 0, 0, 0, 0, 0, 0, K_UT_T / m_T, K_LT_T / m_T, - (K_UT_T + K_LT_T + K_
     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, K_T_se / m_se, -(K_T_se + K_se) / m_se, 0, 0, 0, C_
```

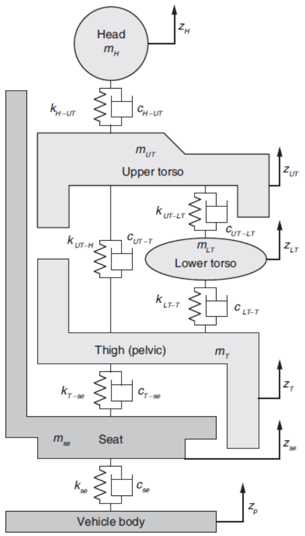
- $\dot{x} = Ax + Bu$
- $y = Cx + Du$

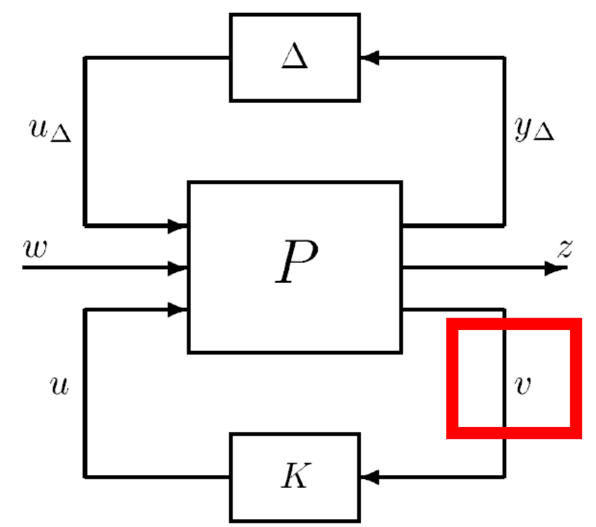
$$u^T = [\, w_{sf}, w_{sr} \,, f_{sf} \,, f_{sr} \,]$$

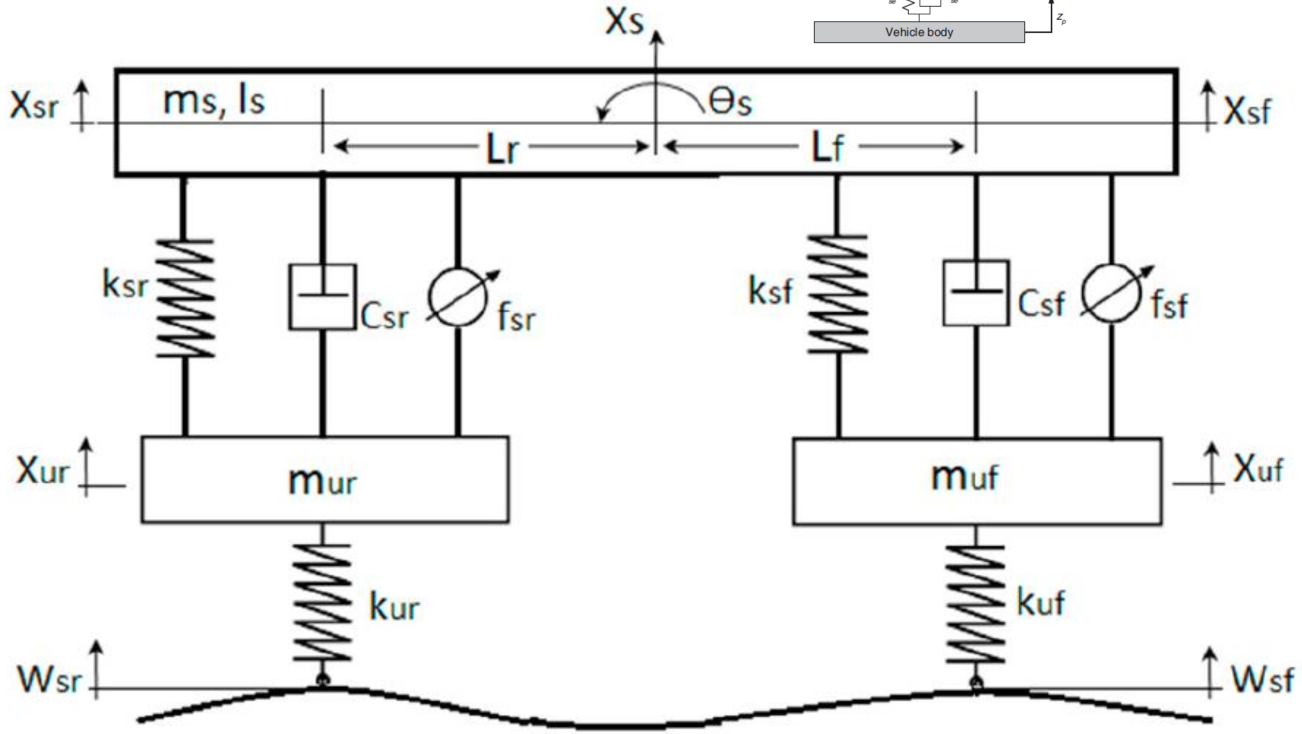
$$y^T = [\, (x_{sf} - x_{uf}) \,, (x_{sr} - x_{ur}), \ddot{z_H}]$$



- Include 5 parts
  - Change output vector v



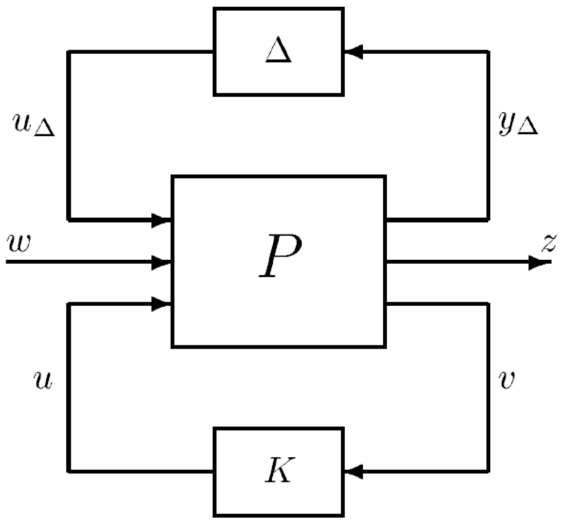




State-Space

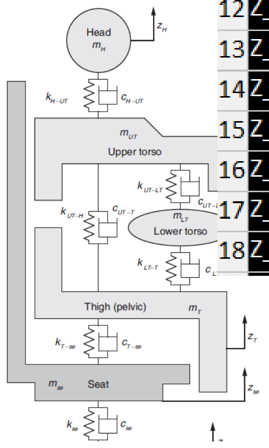
Plant P

- $\dot{x} = Ax + Bu$
- $y = Cx + Du$



$x =$

	hcar.StateName
	1
1	v_sf
2	v_uf
3	v_sr
4	v_ur
5	x_sf
6	x_uf
7	x_sr
8	x_ur
9	Z_H
10	Z_UT
11	Z_LT
12	Z_T
13	Z_se
14	Z_H_dot
15	Z_UT_dot
16	Z_LT_dot
17	Z_T_dot
18	Z_se_dot

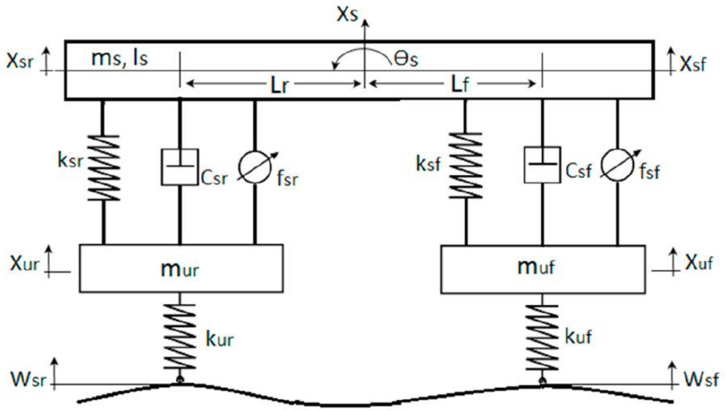


$$u^T = [\, w_{sf}, w_{sr}\,, f_{sf}\,, f_{sr}\,]$$

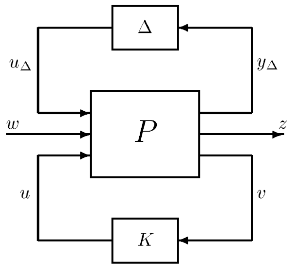
$$y^T = [\,(x_{sf} - x_{uf})\,, (x_{sr} - x_{ur}), \ddot{z}_H]$$

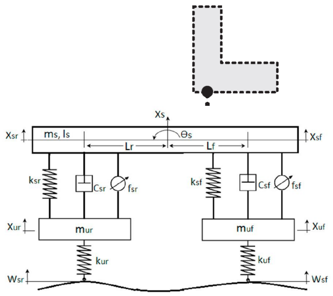
$$z^T = [\,(x_{sf} - x_{uf})\,, (x_{sr} - x_{ur})\,]$$

$$v^T = [\,\ddot{z}_H\,]$$



Uncertainties

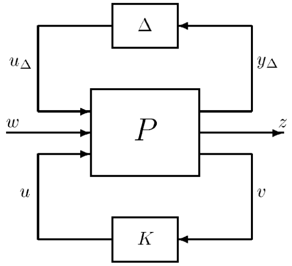


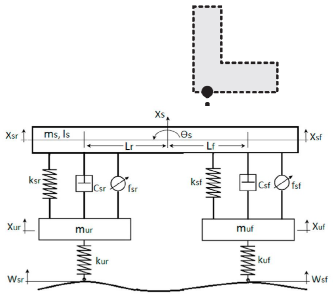


Uncertain continuous-time state-space model with 3 outputs, 4 inputs, 22 states.
The model uncertainty consists of the following blocks:

C_H_UT: Uncertain real, nominal = 310, variability = [-15,15]%, 1 occurrences
C_LT_T: Uncertain real, nominal = 330, variability = [-15,15]%, 1 occurrences
C_T_se: Uncertain real, nominal = 2.48e+03, variability = [-15,15]%, 1 occurrences
C_UT_LT: Uncertain real, nominal = 200, variability = [-15,15]%, 1 occurrences
C_UT_T: Uncertain real, nominal = 909, variability = [-15,15]%, 1 occurrences
C_se: Uncertain real, nominal = 150, variability = [-15,15]%, 1 occurrences
Delta_act1: Uncertain 1x1 LTI, peak gain = 1, 1 occurrences
Delta_act2: Uncertain 1x1 LTI, peak gain = 1, 1 occurrences
K1: Uncertain real, nominal = 1.08, variability = [-10,10]%, 1 occurrences
K2: Uncertain real, nominal = 1.08, variability = [-10,10]%, 1 occurrences
T1: Uncertain real, nominal = 0.005, variability = [-20,20]%, 1 occurrences
T2: Uncertain real, nominal = 0.005, variability = [-20,20]%, 1 occurrences

Uncertainties





Uncerta **Dampers in the Passenger Model** 3 outputs, 4 inputs, 22 states.

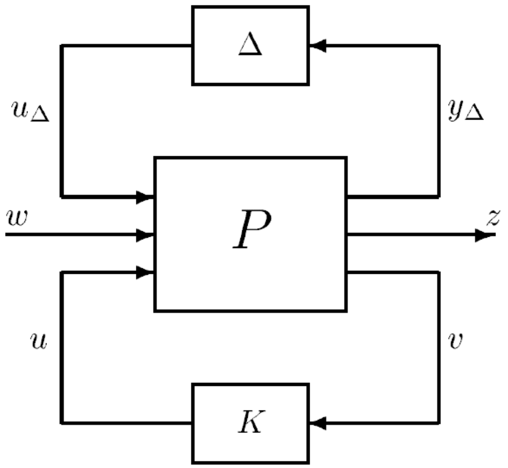
The model uncertainty consists of the following blocks:

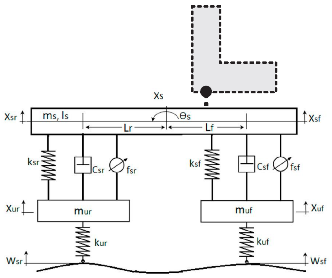
C_H_UT: Uncertain real, nominal = 310, variability = [-15,15]%, 1 occurrences
C_LT_T: Uncertain real, nominal = 330, variability = [-15,15]%, 1 occurrences
C_T_se: Uncertain real, nominal = 2.48e+03, variability = [-15,15]%, 1 occurrences
C_UT_LT: Uncertain real, nominal = 200, variability = [-15,15]%, 1 occurrences
C_UT_T: Uncertain real, nominal = 909, variability = [-15,15]%, 1 occurrences
C_se: Uncertain real, nominal = 150, variability = [-15,15]%, 1 occurrences
Delta_act1: Uncertain 1x1 LTI, peak gain = 1, 1 occurrences
Delta_act2: Uncertain 1x1 LTI, peak gain = 1, 1 occurrences
K1: Uncertain real, nominal = 1.08, variability = [-10,10]%, 1 occurrences
K2: Uncertain real, nominal = 1.08, variability = [-10,10]%, 1 occurrences
T1: Uncertain real, nominal = 0.005, variability = [-20,20]%, 1 occurrences
T2: Uncertain real, nominal = 0.005, variability = [-20,20]%, 1 occurrences

Uncertainties in Two Hydraulic Actuators

μ synthesis – Part 1

- Robust Performance



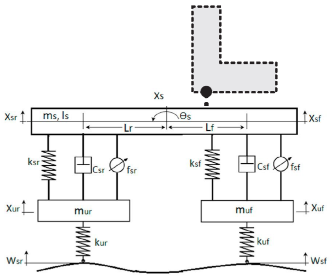


D-K ITERATION SUMMARY:

	Robust performance			Fit order
Iter	K Step	Peak MU	D Fit	D
1	2.46	0.2235	0.2278	42
2	0.2278	0.2256	0.2279	58
3	0.2279	0.2256	0.2278	56

Best achieved robust performance: 0.223

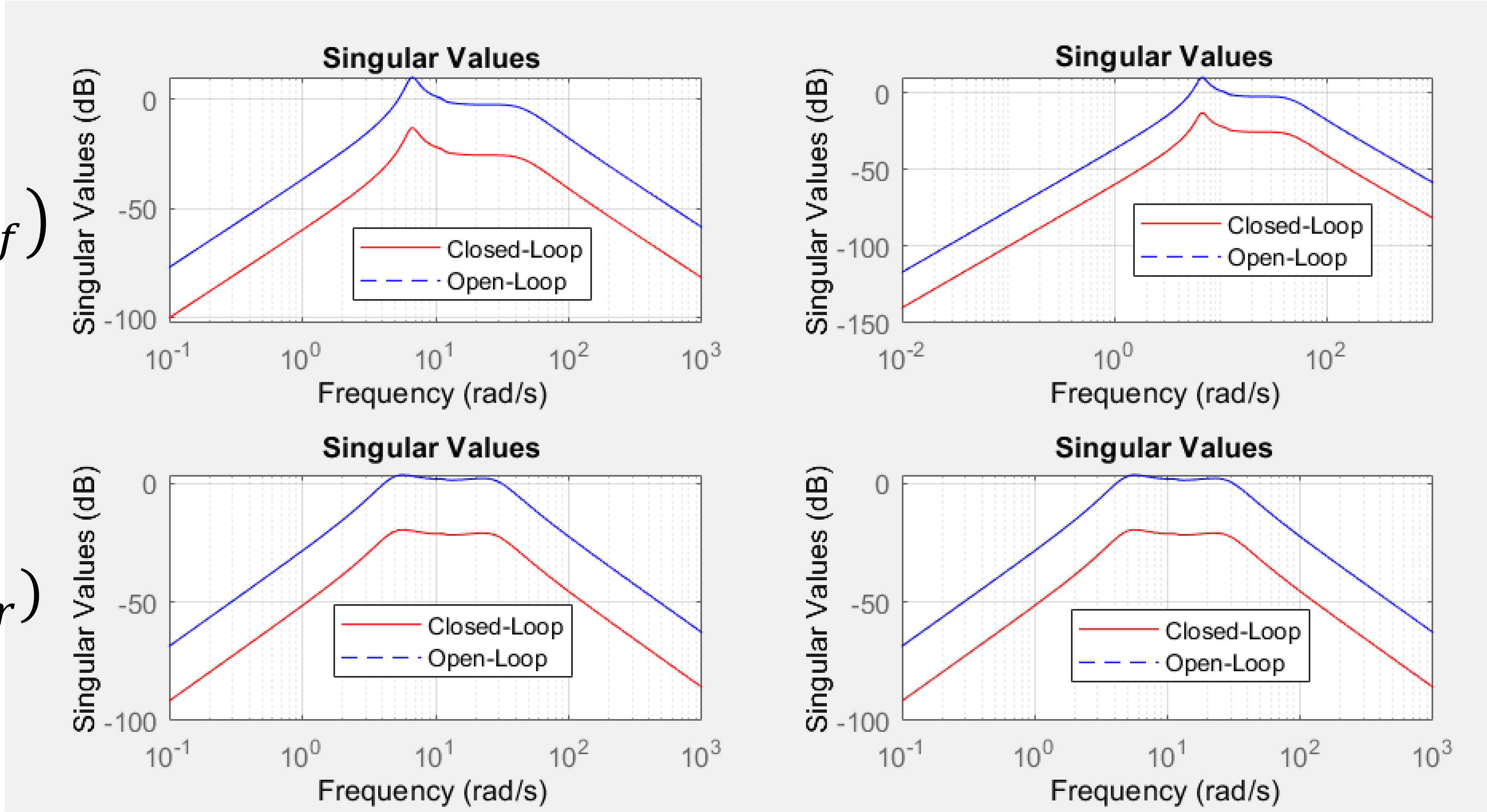
µ synthesis – Part 1



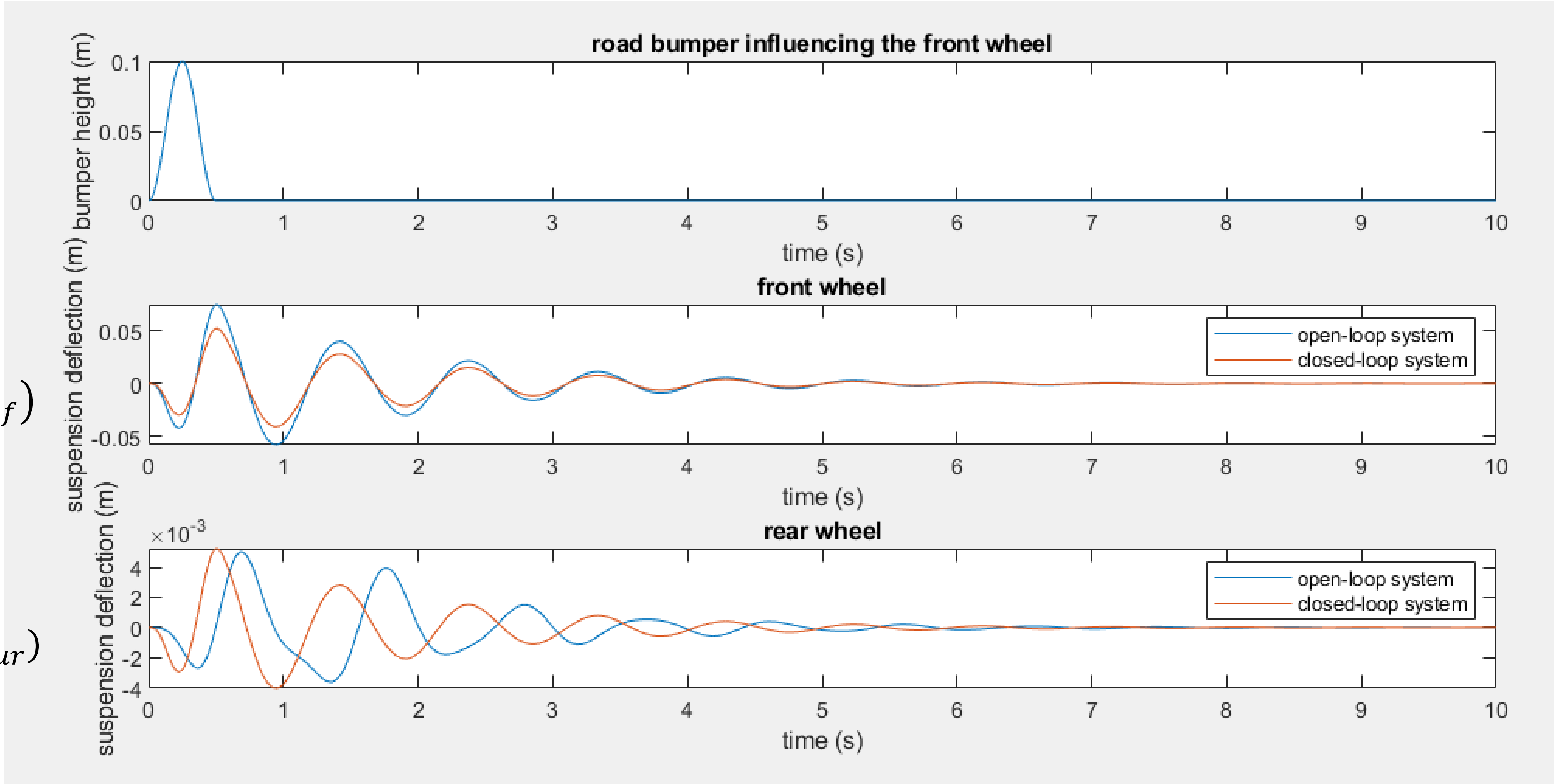
From w_{sf} From w_{sr}

To $(x_{sf} - x_{uf})$

To $(x_{sr} - x_{ur})$



μ synthesis – Part 1



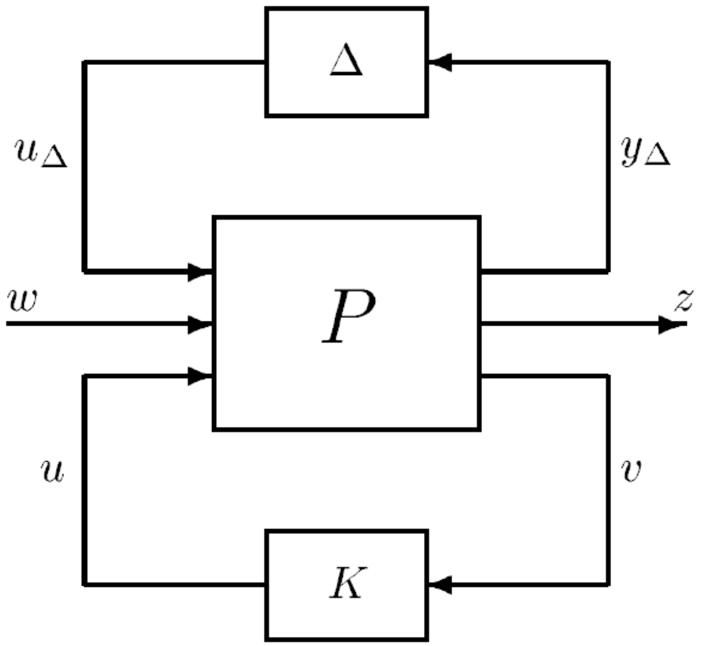
$(x_{sf} - x_{uf})$

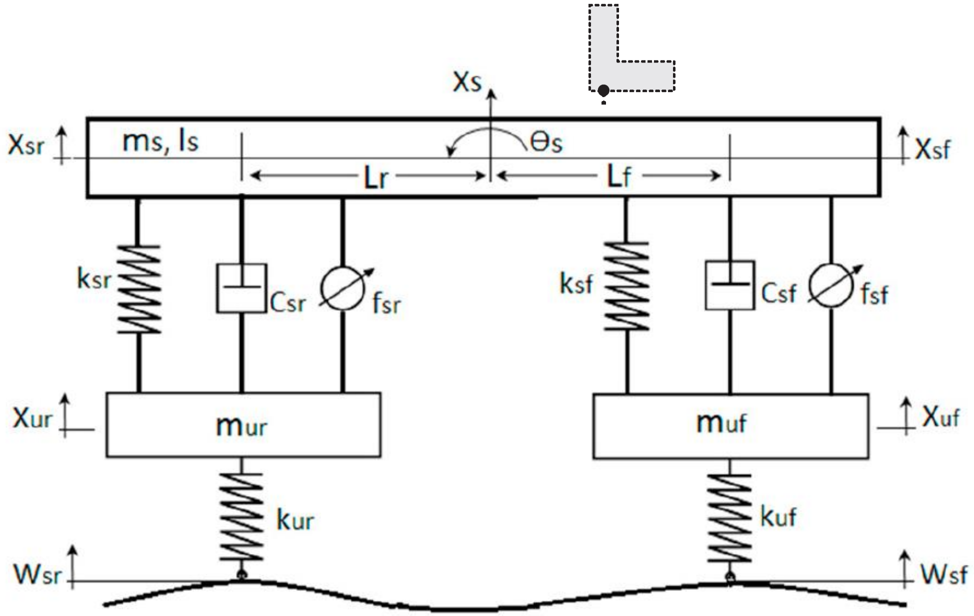
$(x_{sr} - x_{ur})$

μ synthesis – Part 2

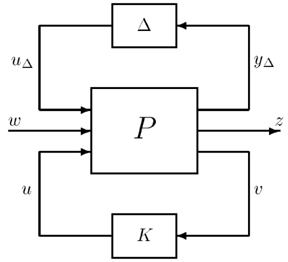
$$z^T = [\,(x_{sf} - x_{uf})\,, (x_{sr} - x_{ur})\,]$$

$$v^T = [\,\ddot{x}_s\,]$$





μ synthesis – Part 2



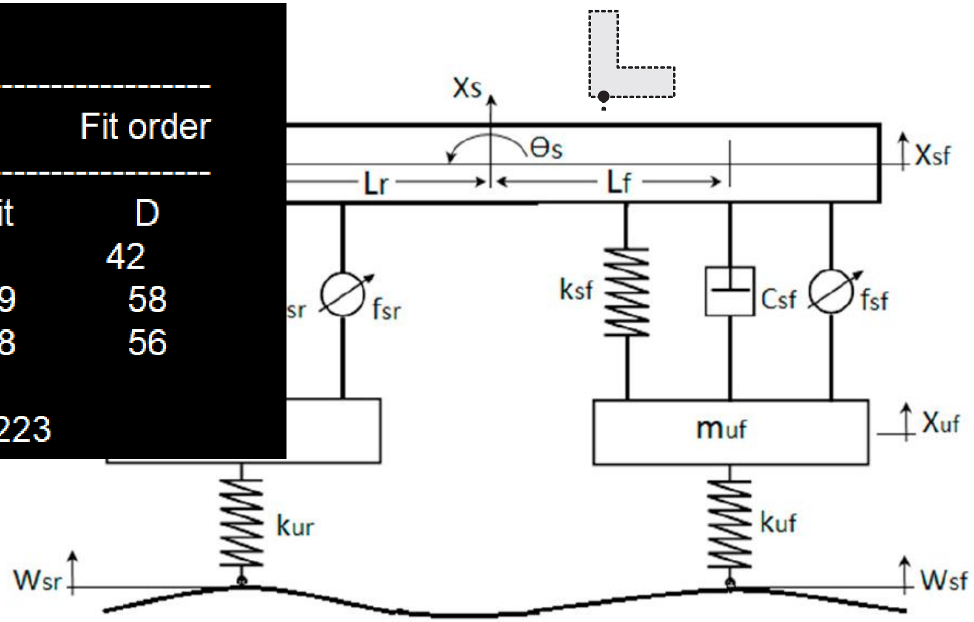
- Robust Performance

$$z^T = [\,(x_{sf} - x_{uf})\,,\,(x_{sr} - x_{ur})\,]$$
$$v^T = [\,\ddot{x}_s\,]$$

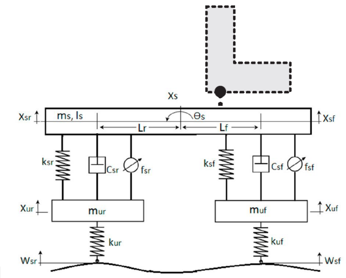
D-K ITERATION SUMMARY:

	Robust performance			Fit order
Iter	K Step	Peak MU	D Fit	D
1	2.46	0.2235	0.2278	42
2	0.2278	0.2256	0.2279	58
3	0.2279	0.2256	0.2278	56

Best achieved robust performance: 0.223



µ synthesis – Part 2

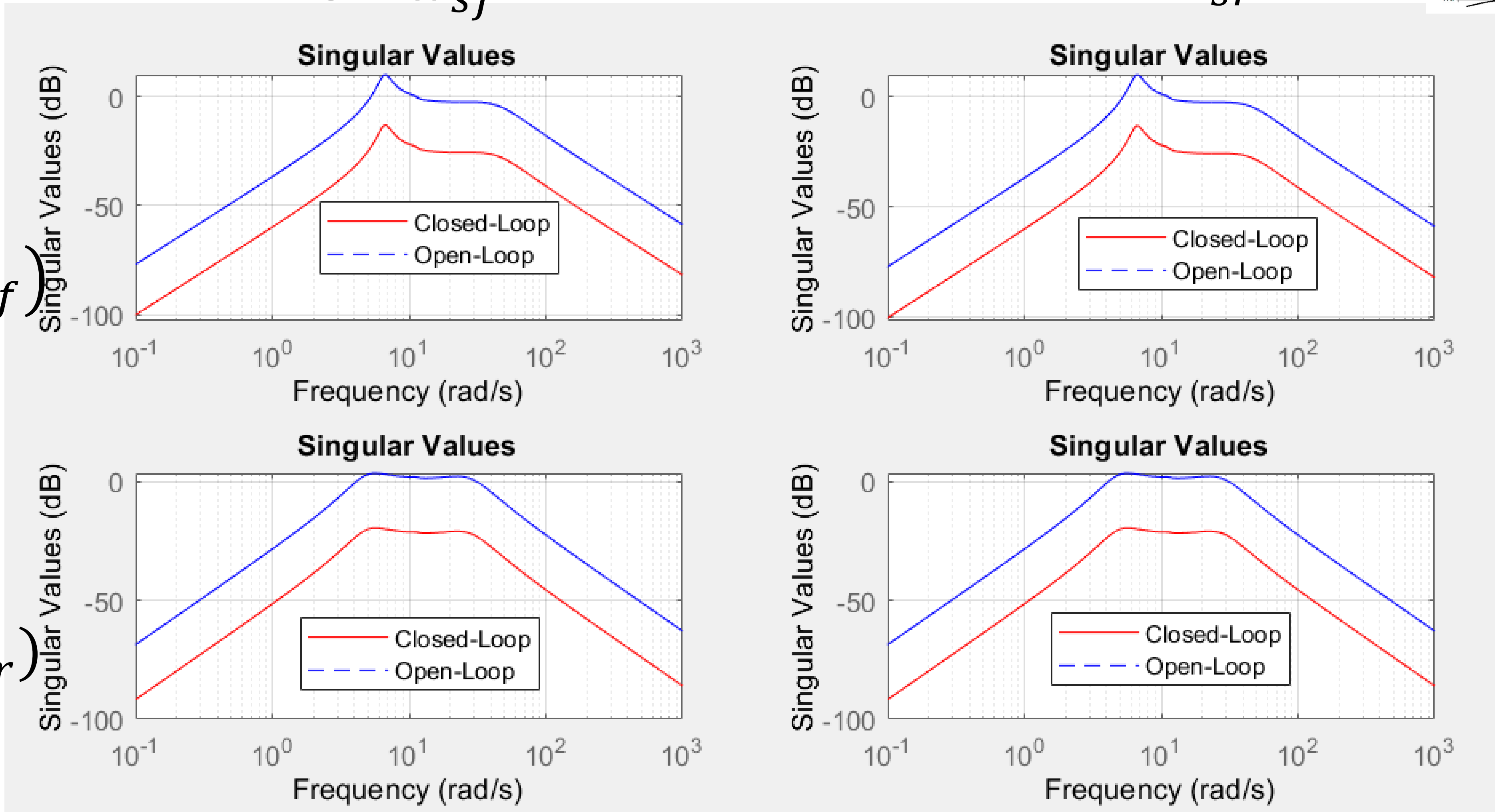


From w_{sf}

From w_{sr}

To $(x_{sf} - x_{uf})$

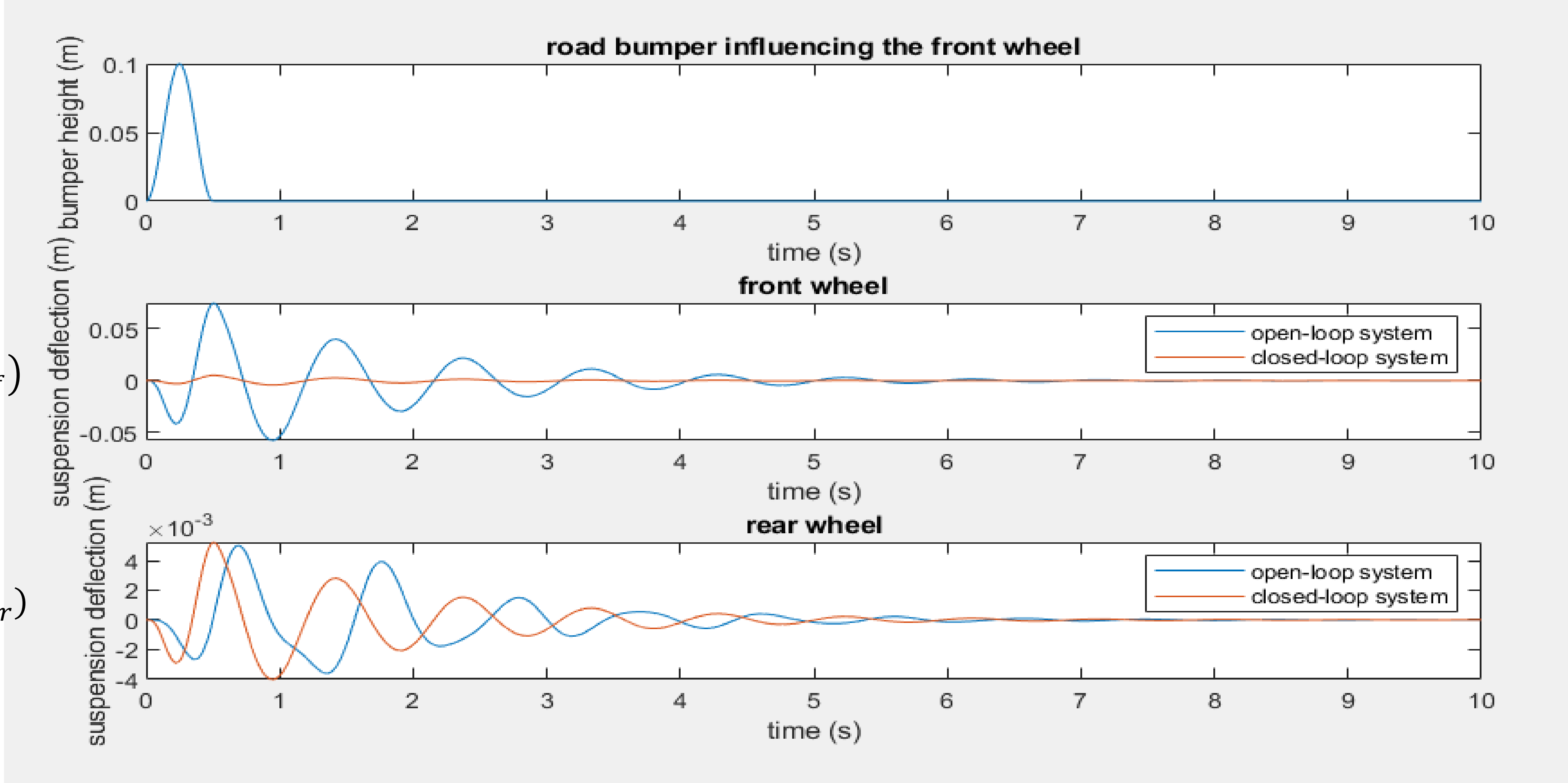
To $(x_{sr} - x_{ur})$



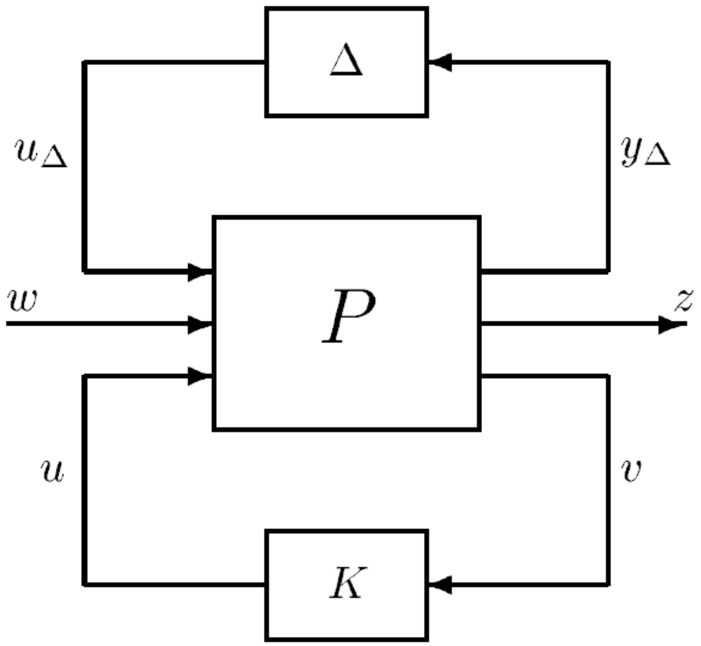
μ synthesis – Part 2

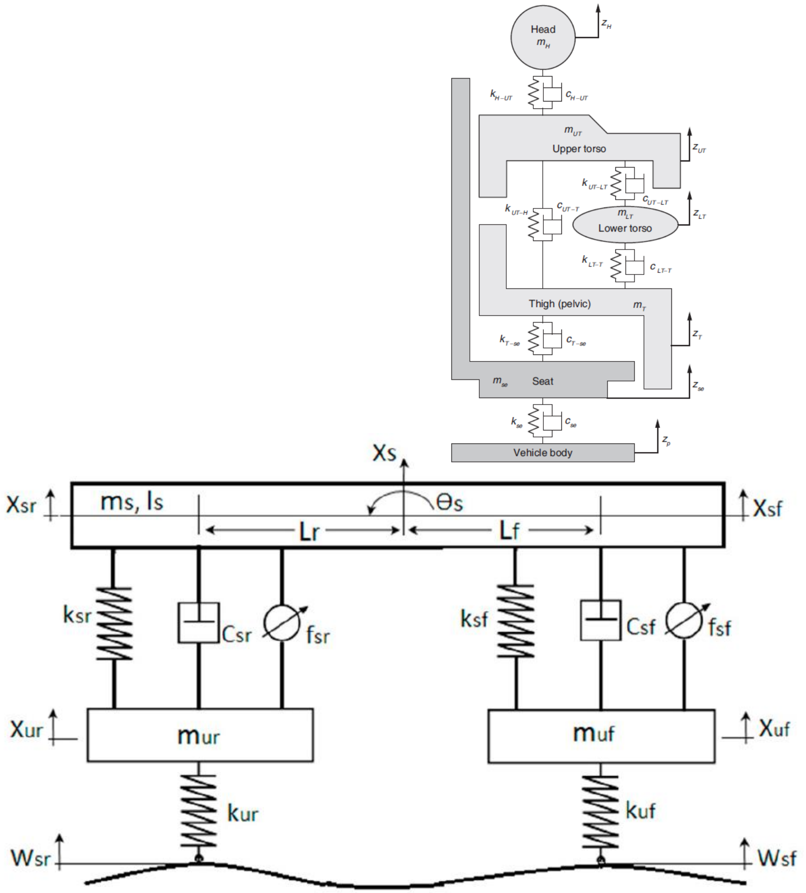
$(x_{sf} - x_{uf})$

$(x_{sr} - x_{ur})$



μ synthesis – Part 3





$$z^T = [\,(x_{sf} - x_{uf}),\,(x_{sr} - x_{ur})\,]$$

$$v^T = [\ddot{x}_s\,,\ \ddot{Z}_H\,]$$

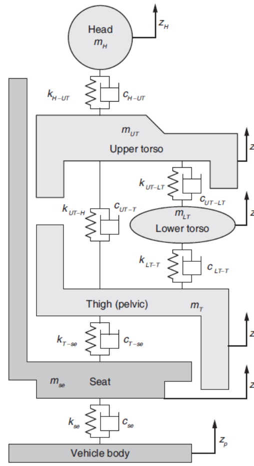
μ synthesis – Part 3

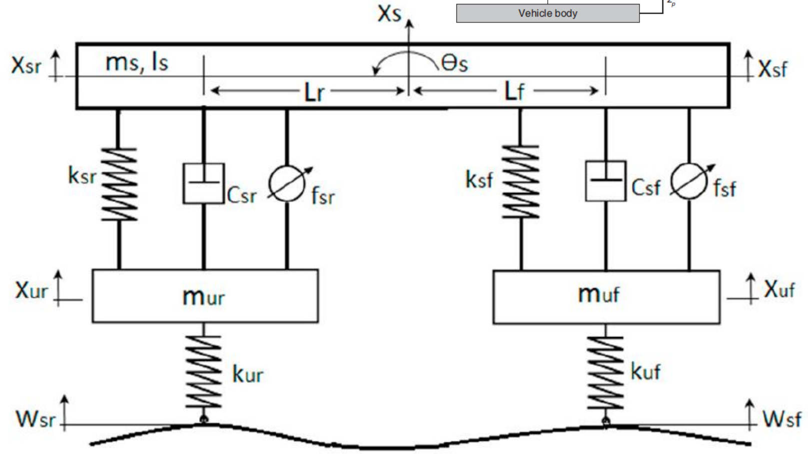
D-K ITERATION SUMMARY:

		Robust performance		Fit order
Iter	K Step	Peak MU	D Fit	D
1	2.46	0.2235	0.2277	40
2	0.2277	0.2256	0.2277	62
3	0.2277	0.2256	0.2278	50

Best achieved robust performance: 0.223

order





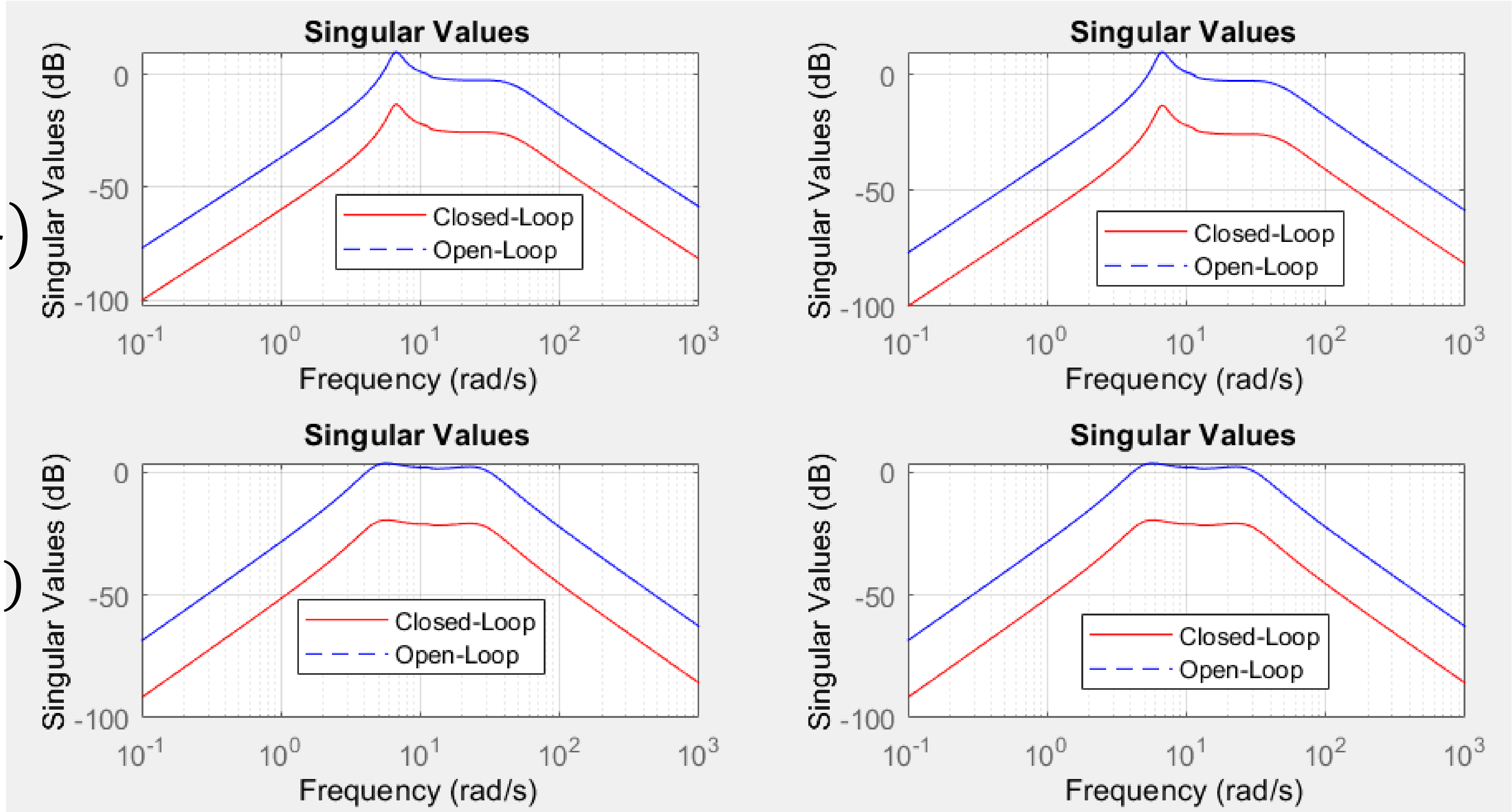
$$z^T = [\,(x_{sf} - x_{uf})\,, (x_{sr} - x_{ur})\,]$$
$$v^T = [\ddot{x}_s\,,\ \ddot{Z}_H\,]$$

μ synthesis – Part 3

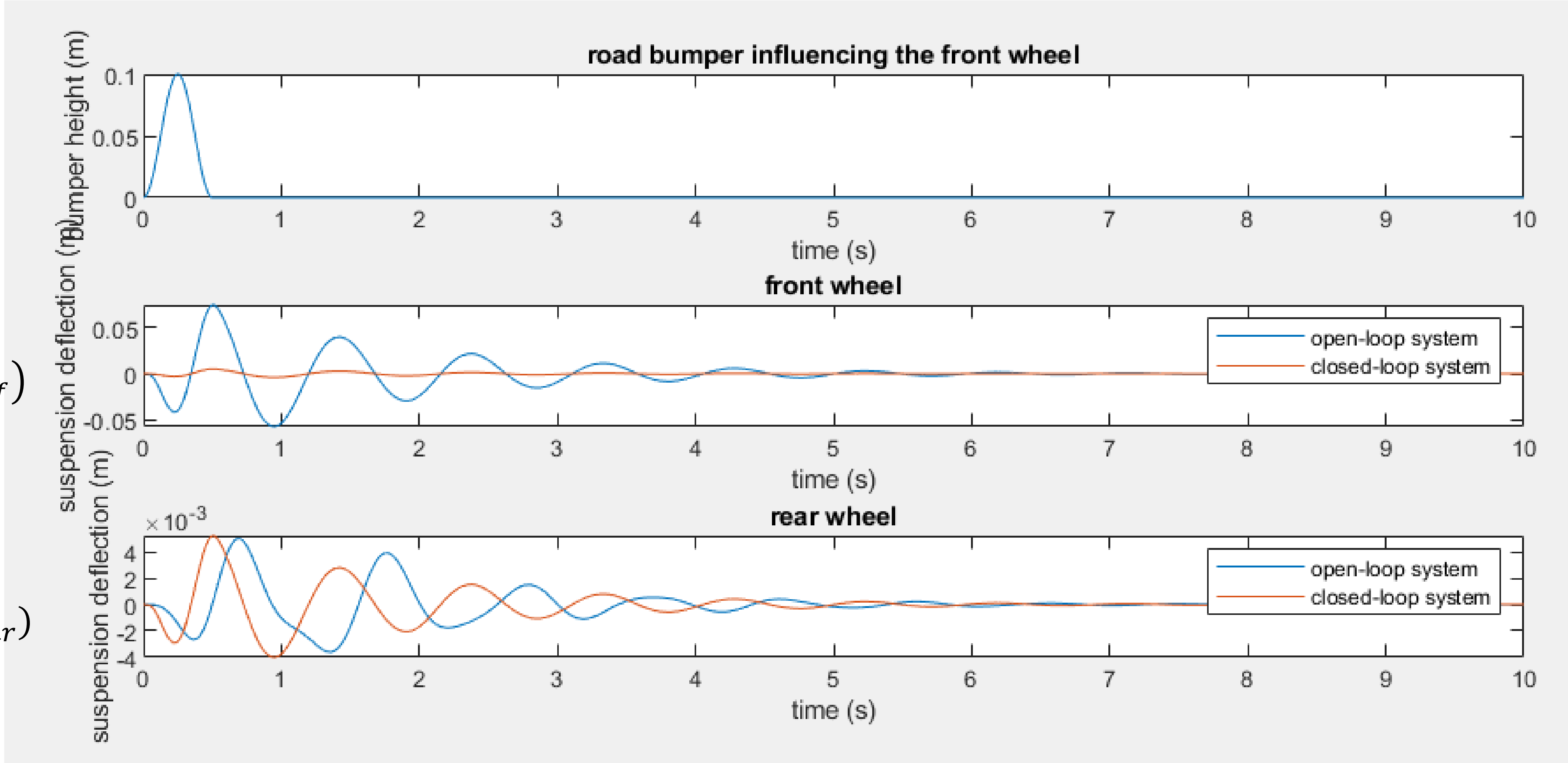
From w_{sf} | From w_{sr}

To $(x_{sf} - x_{uf})$

To $(x_{sr} - x_{ur})$



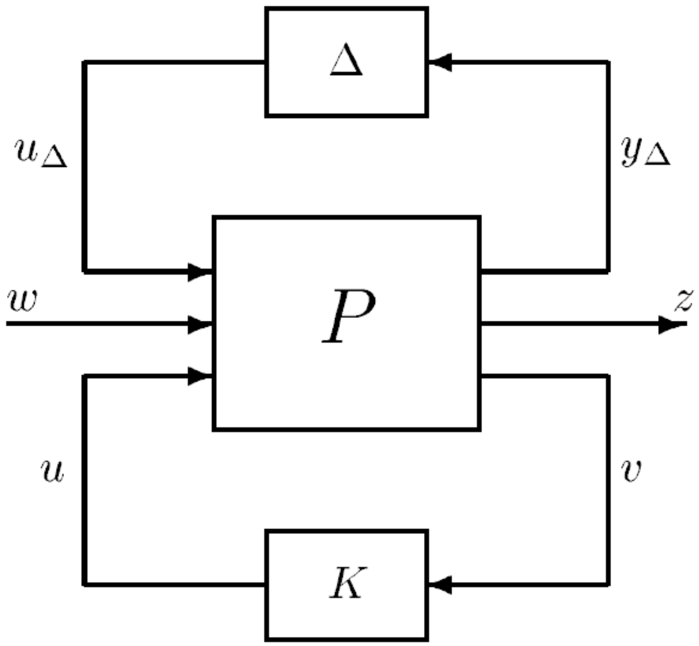
μ synthesis – Part 3

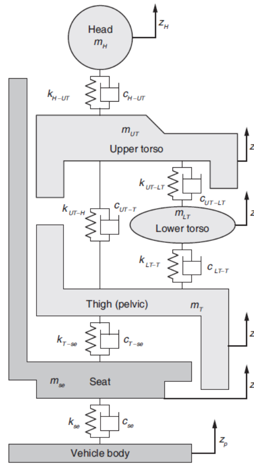


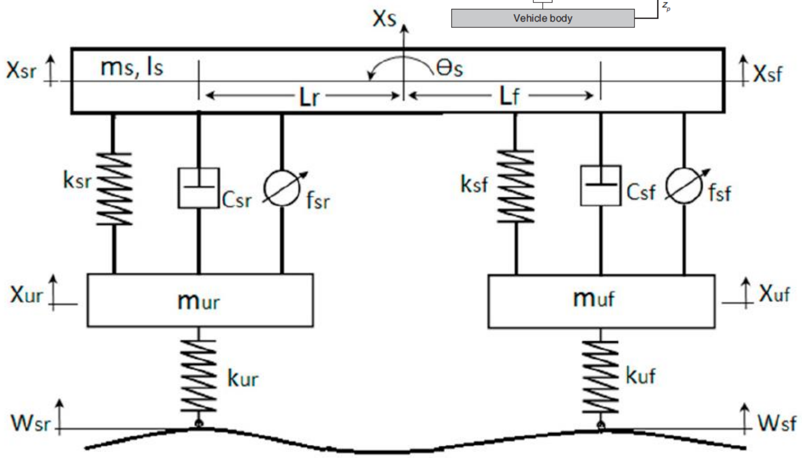
$(x_{sf} - x_{uf})$

$(x_{sr} - x_{ur})$

μ synthesis – Part 4







$$z^T = [\,(x_{sf} - x_{uf})\,,(x_{sr} - x_{ur}),\,\ddot{x}_s\,,\ \ddot{Z}_H\,]$$

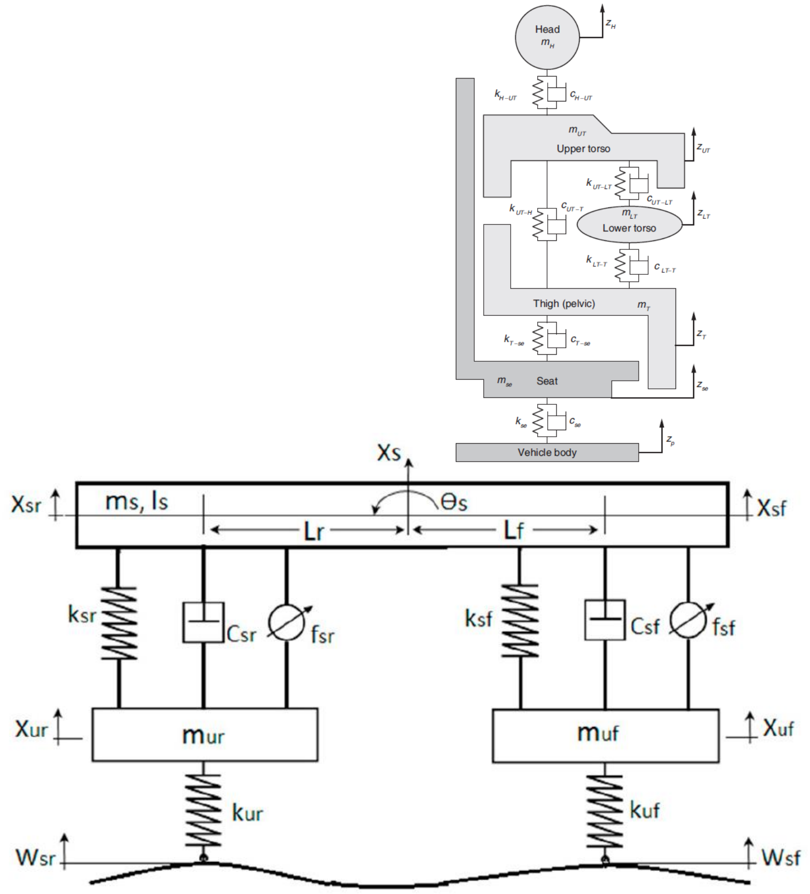
$$v^T = [\ddot{x}_s\,,\ \ddot{Z}_H\,]$$

μ synthesis – Part 4

D-K ITERATION SUMMARY:

	Robust performance			Fit order
Iter	K Step	Peak MU	D Fit	D
1	26.1	25.8	26.04	8
2	26.04	25.8	26.09	20
3	26.09	25.8	26.09	12

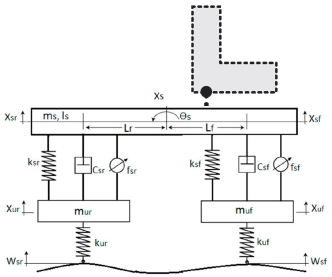
Best achieved robust performance: 25.8



$$z^T = [\,(x_{sf} - x_{uf})\,,\,(x_{sr} - x_{ur}),\,\ddot{x}_s\,,\ \ddot{Z}_H\,]$$

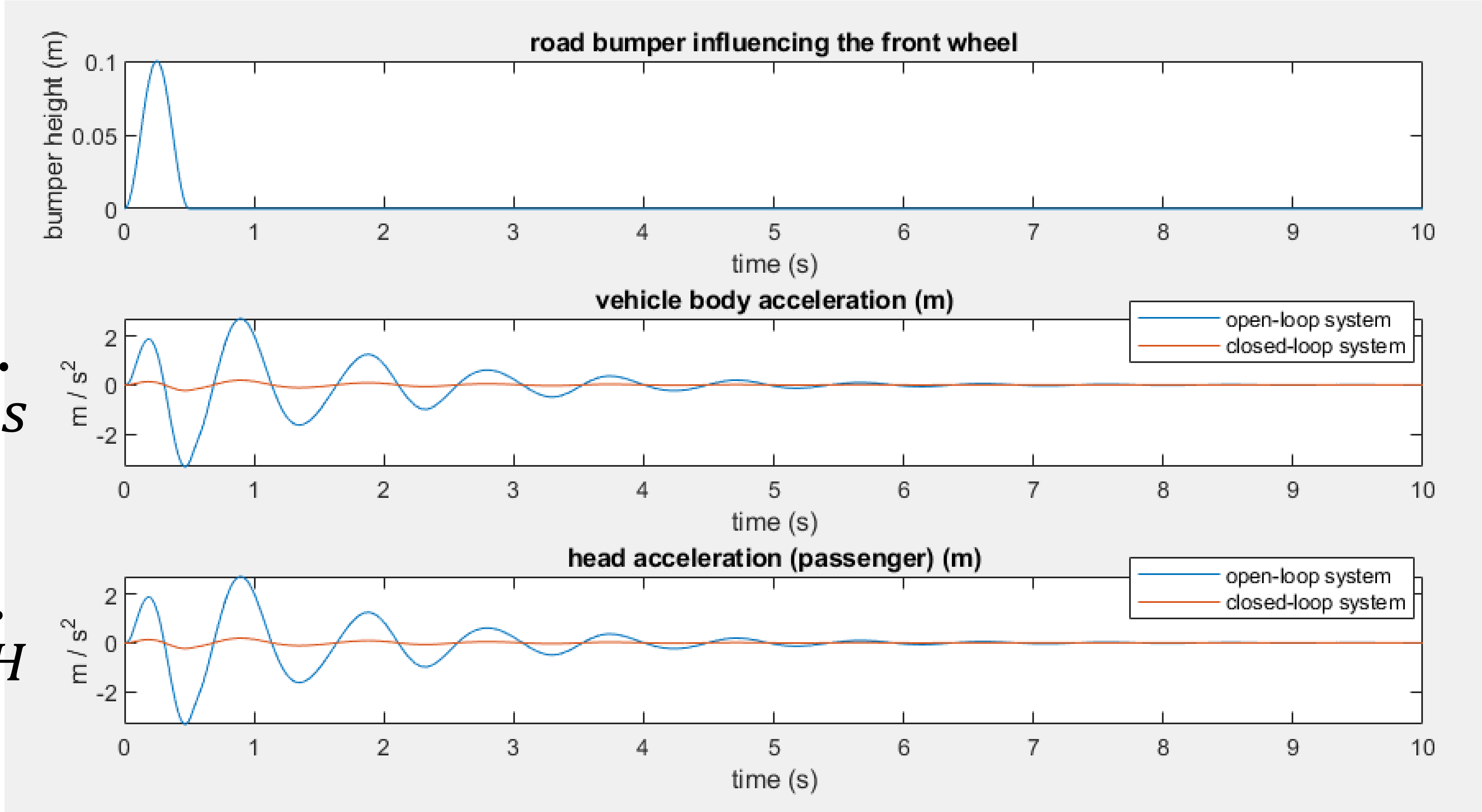
$$v^T = [\ddot{x}_s\,,\ \ddot{Z}_H\,]$$

μ synthesis – Part 4

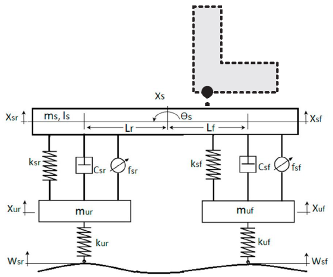


$\ddot{x}_s$

$\ddot{z}_H$



μ synthesis – Part 4 (samples = 100)

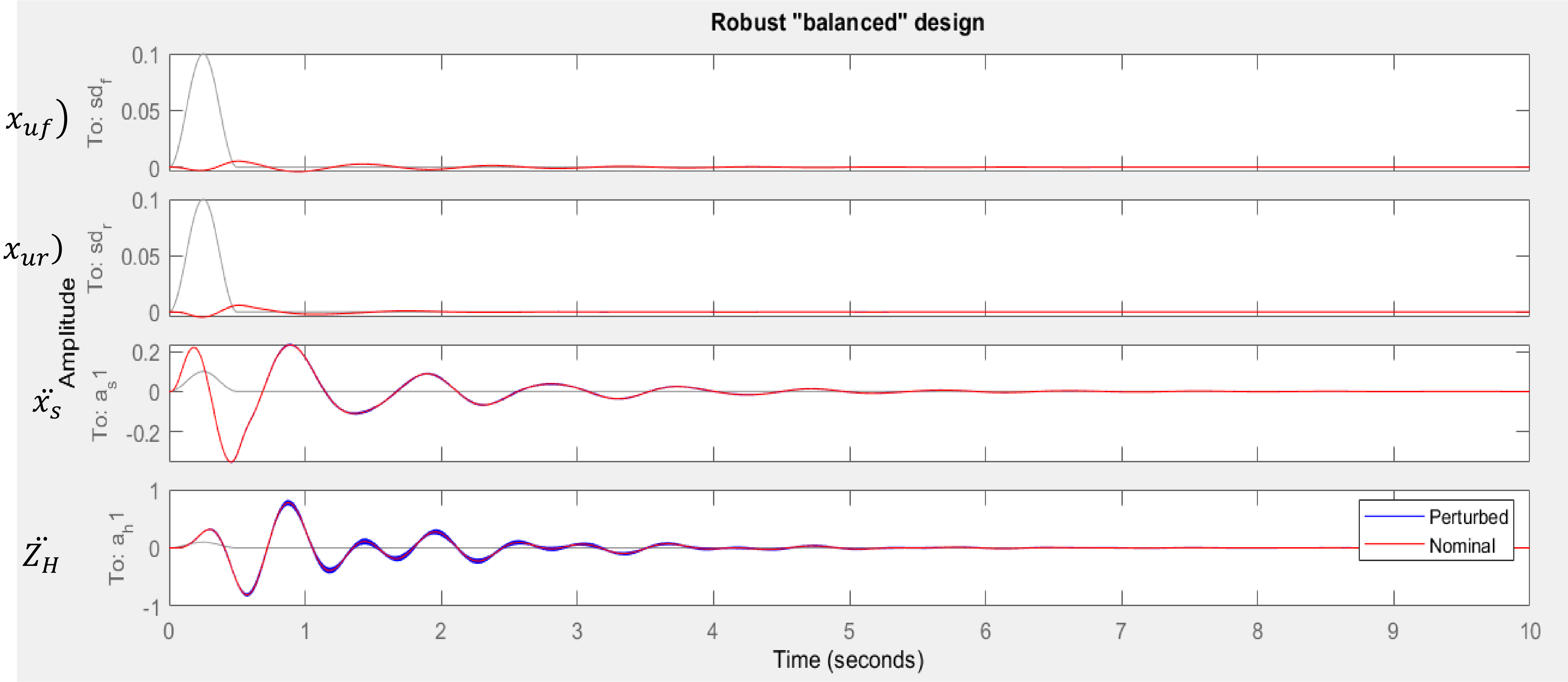


$(x_{sf} - x_{uf})$

$(x_{sr} - x_{ur})$

$\ddot{x}_s$

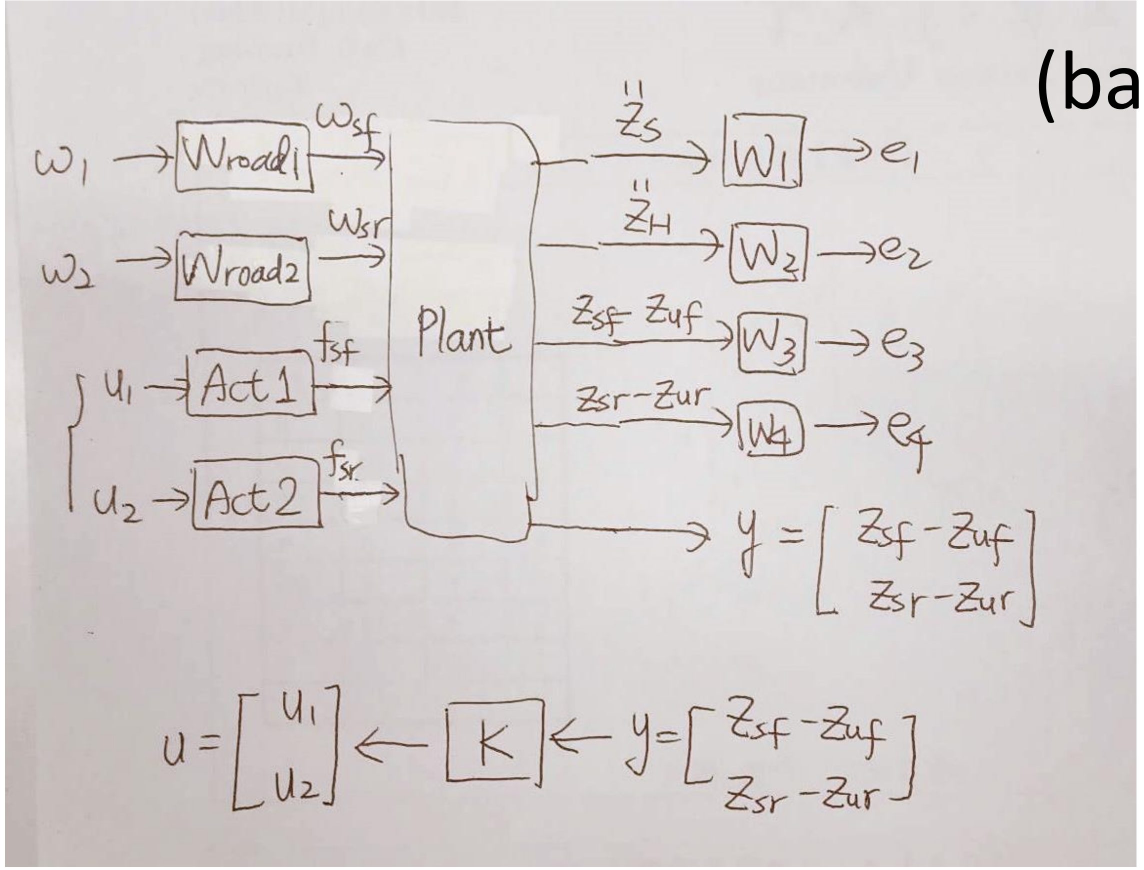
$\ddot{Z}_H$



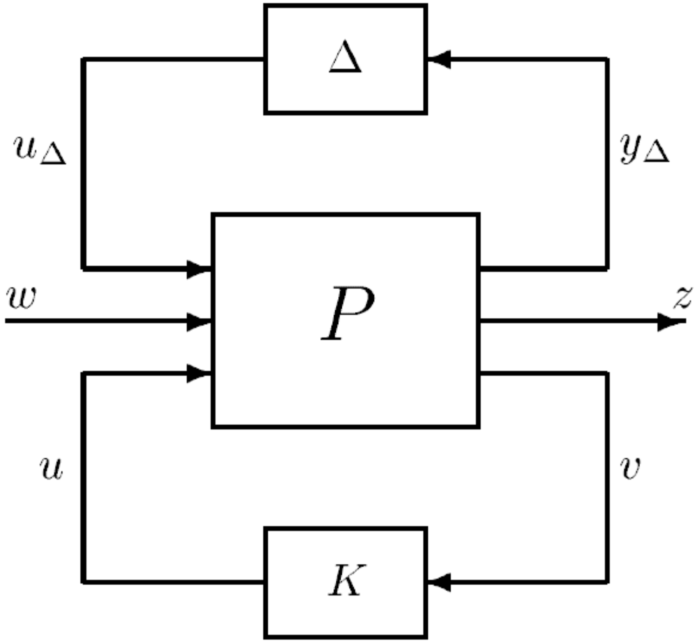
μ synthesis – Best Case

$$W_1(s) = \frac{0.72s}{s^2 + 7.2s + 5200}$$

(band-pass filter)



μ synthesis – Best Case



$$z^T = [\ddot{x}_s\,,\ddot{Z}_H\,,(x_{sf} - x_{uf})\,,(x_{sr} - x_{ur})\,]$$

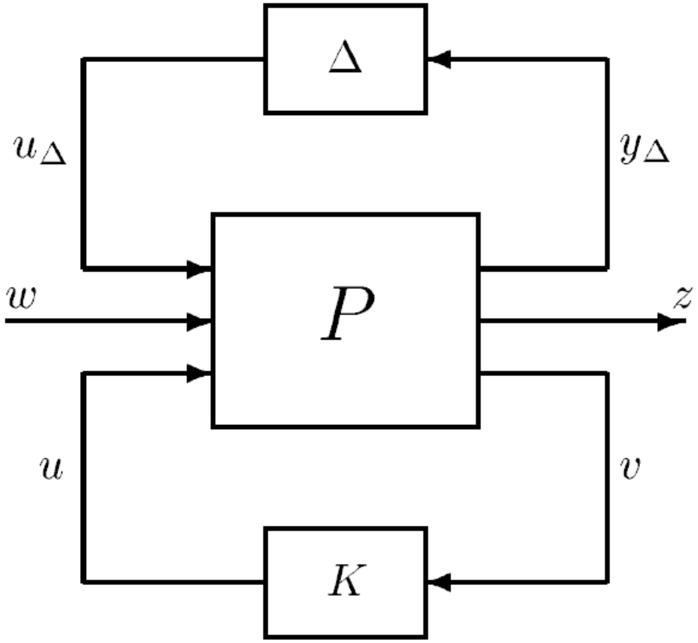
$$v^T = [(x_{sf} - x_{uf})\,,(x_{sr} - x_{ur})]$$

μ synthesis – Best Case

D-K ITERATION SUMMARY:

Iter	K Step	Peak MU	D Fit	D
	Robust performance			Fit order
1	2.46	0.301	0.3036	66
2	0.3036	0.301	0.3026	62
3	0.3026	0.301	0.3024	62

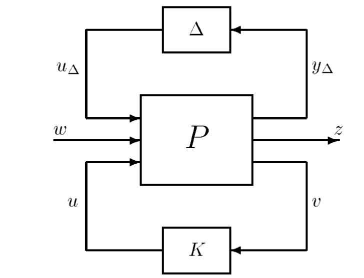
Best achieved robust performance: 0.301



$$z^T = [\ddot{x}_s , \ddot{Z}_H , (x_{sf} - x_{uf}) , (x_{sr} - x_{ur})]$$

$$v^T = [(x_{sf} - x_{uf}) , (x_{sr} - x_{ur})]$$

µ synthesis – Best Case

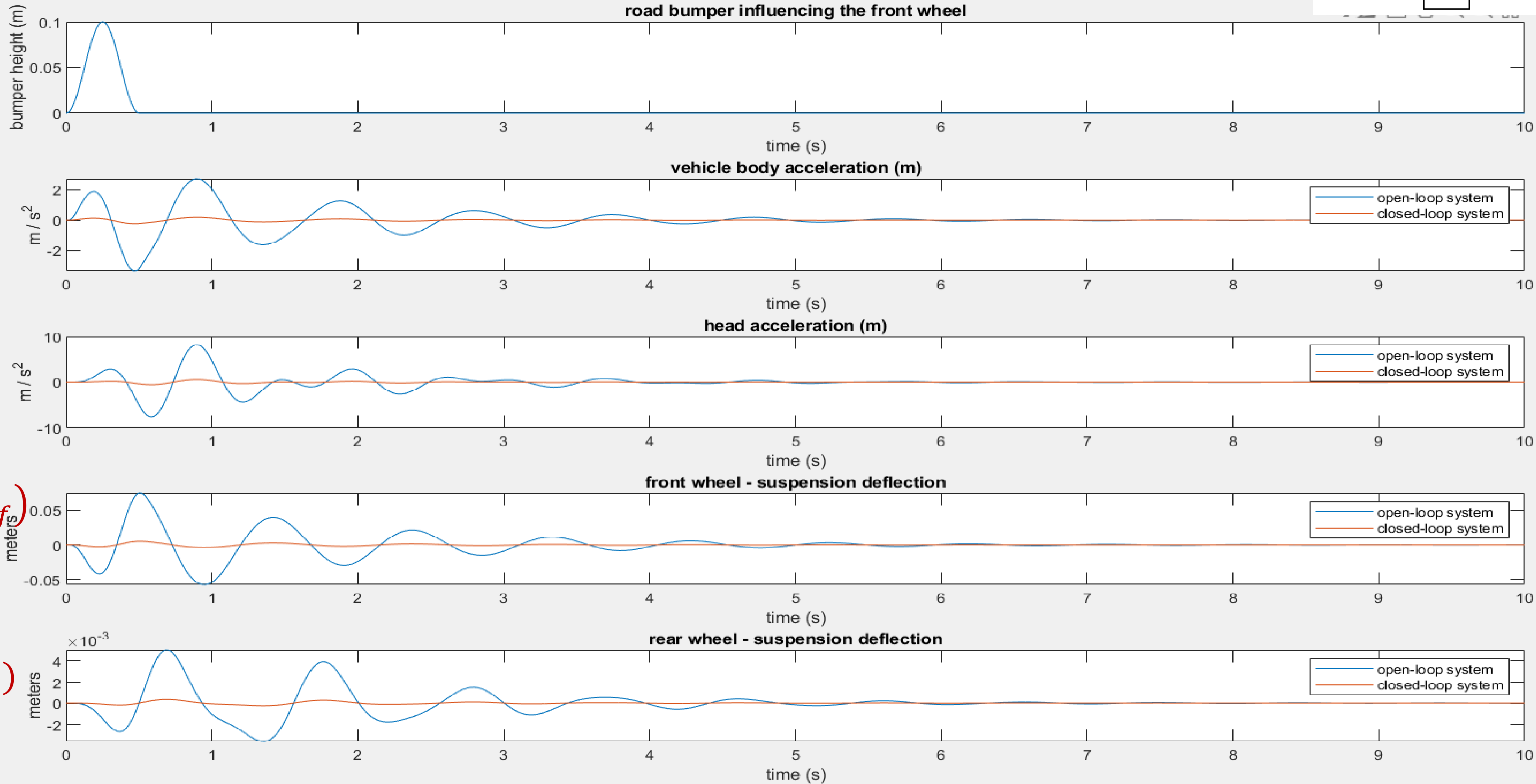


$\ddot{x}_s$

$\ddot{Z}_H$

$(x_{sf} - x_{uf})$

$(x_{sr} - x_{ur})$



Conclusion

- achieves robust performance
- improves ride comfort
- improves suspension deflections

Conclusion

- This project establishes a robust feedback control synthesis for a class of half-car suspension systems considering a 4-DOF passenger's biodynamics with parametric uncertainties.

Reference

1. Gandhi, Puneet & Sasidharan, Adarsh & Ramachandran, K.I.. (2017). Performance Analysis of Half Car Suspension Model with 4 DOF using PID, LQR, FUZZY and ANFIS Controllers. Procedia Computer Science. 115. 2-13. 10.1016/j.procs.2017.09.070.
2. Gudarzi M, Oveisi A. Robust Control for Ride Comfort Improvement of an Active Suspension System considering Uncertain Driver's Biodynamics. Journal of Low Frequency Noise, Vibration and Active Control. 2014;33(3):317-339. doi:10.1260/0263-0923.33.3.317
3. Jibril, Mustefa (2020). H ∞ and μ-synthesis Design of Quarter Car Active Suspension System. *International Journal of Scientific Research and Engineering Development* 3 (1):608-619. PhilArchive copy v1: https://philarchive.org/archive/JIBHAv1

Thanks for listening